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Full Discretization Scheme for the Dynamics of Elliptic Membrane Shell Model

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Abstract. In this study, the dynamics of elliptic membrane shell model has been proposed and discussed numerically for the first time. Firstly, we show that the solution of this model exists and is unique. Secondly, we consider spatial and time discretizations of the time-dependent elliptic membrane shell by finite element method and Newmark scheme, respectively. Then, the corresponding existence, uniqueness, stability, convergence and a priori error estimate are given. Finally, we present numerical results involving a portion of an ellipsoidal shell and a portion of a spherical shell to verify the efficiency and convergence of the numerical scheme.

AMS subject classifications: 65N12, 65N15, 65N30

Key words: Elliptic membrane shell, finite element method, full discretization, ellipsoidal shell, spherical shell.

1 Introduction

Elliptic shells, such as ellipsoidal shells and spherical shells, have been applied in many fields, such as biological engineering (cf. [1, 2]), composite material (cf. [3]), artificial intelligence (cf. [4]), the construction of nuclear power plants building (cf. [5]), magnetic industry (cf. [6, 7]), and marine structures like egg-shaped pressure hull (cf. [8]). The theory of elliptic membrane shells is one of the most important branches in elastic shells theory.

In 1973, the theory of shells and plates was proposed in [9]. The mathematical justification for membrane shells model and generalized membrane shells model was given,

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respectively, in [10] and [11]. The error estimates for the membrane shells model and the analysis of generalized membrane shell with hyperbolic [12] and parabolic [13] middle surfaces were given, respectively. Ciarlet [14] provided a detailed introduction on elliptic membrane shell including definitions, theories and examples. The elliptic membrane shell is defined by means of its middle surface, which is required to be uniformly elliptic and clamped along the entire boundary. Later, the Donati compatibility conditions for membrane shells through the intrinsic approach, i.e., a quadratic minimization problem with the linearized change of metric and change of curvature tensors of the middle surface of the shell as the new unknowns, were discussed in [15].

For the difficult problem of dynamic membrane shells, which can be equivalently formulated by means of a set of hyperbolic equations, the literature related to its theory or model is quite sparse. For viscoelastic elliptic membrane shell, there are also very few theoretical results. In [16], for instance, the authors justified the two-dimensional equations of a viscoelastic elliptic membrane shell, which includes a long-term memory that takes into account previous deformations. In [17], Dong et al. provided a multi-scale computational method for dynamic thermo-mechanical performance of heterogeneous shell structures.

To the best of our knowledge, there is rare literature [18] related to the numerical simulation of the time-dependent model involving membrane shells, which is based on the steady generalized membrane shell model proposed in [14]. The study of the more general time-dependent Koiter's model has been discussed in [19] and [20]. Compared with the generalized membrane shell, the main numerical difficulties for the elliptic membrane shell model is that the geometric hypothesis and boundary conditions are more strict.

In this paper, we study the time-dependent elliptic membrane shell model. Firstly, we established the time-dependent elliptic membrane shell model and proved the existence and uniqueness of the solution of the model. Secondly, the spatial variable is discretized by means of the finite element method and the full time discretization of the model is performed using the Newmark scheme. Then the corresponding results of existence, uniqueness, stability, convergence and a priori error estimate are given. These results rely on some technical approximation hypotheses for which we provide some references. Finally, we provided numerical experiments for a portion of an ellipsoidal shell and a portion of a spherical shell. In addition, the stability and validity of the model are verified by the calculation of the convergence order and error of the ellipsoid shells with different time steps and space steps.

2 Preliminaries

Our notation is essentially borrowed from [14]. In what follows, Latin indices and exponents take their values in the set $\{1,2,3\}$, whereas Greek indices and exponents take their values in the set $\{1,2\}$. In addition, the repeated index summation convention is systematically used. The Euclidean inner product and the exterior product of any vectors