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On Accurately Resolving Detonation Dynamics by Adaptive Finite Volume Method on Unstructured Grids

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Abstract. Long time simulations are needed in the numerical study of the Zeldovich-Neumann-Döring model, in which the quality resolving the dynamics of the detonation front is crucial. The numerical error introduced from the inappropriate outflow boundary condition and the mesh resolution are two main factors qualitatively affecting the dynamics of the detonation front. In this paper we improve the numerical framework in [15] by introducing the Strang splitting method and a new h-adaptive method with a feature based *a posteriori* error estimator. Then a cheap numerical approach is proposed to sharply estimate a time period, in which the unphysical influence on the detonation front can be avoided effectively. The sufficiently dense mesh resolution can be guaranteed around the detonation front and in the reaction zone by the proposed h-adaptive method. The numerical results show that the proposed method is sufficiently robust even for long time calculations, and the quality dynamics of the detonation can be obtained by the proposed numerical approach.

AMS subject classifications: 65N50, 35Q31

Key words: Reactive Euler equations, Strang splitting scheme, h-adaptive methods, subsonic outflow boundary, ZND model.

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1 Introduction

The time-dependent, nonlinear reactive Euler equations have been playing an important role in the study of the detonation phenomenon. Since its intrinsic instability, in the high dimensional study of the detonation, there would be transverse fluctuation appearing along the detonation front, which can result in very complex dynamics in the following reaction zone. To well understand the mechanics of this unstable phenomenon would benefit the practical application of the detonation such as the design of the rotating detonation engine [23].

Although there have been many pioneer works [27,28] for the stability analysis on the detonation phenomenon, the linearization can not be avoided in most of them. It is the direct numerical simulation a method on fully resolving the nonlinearity of the governing equation, which has motivated the works such as [12, 13, 25, 35] for the finite difference methods, [4] for the finite volume methods, [7] for the finite element methods, [39] for the discontinuous Galerkin methods, etc. However, the advantages mentioned above from the direct numerical simulations can be obtained only if the simulations are quality, i.e., the dynamics of the detonation front is well resolved, and the simulation time is sufficiently long. Hence, it is demanding on the computational resource, especially for high dimensional simulations. To improve the efficiency of the implementation, several acceleration techniques have been applied on the numerical methods. For examples, the fractional time stepping methods in which the convection process and reaction process are treated as two independent processes. By combining these two processes with different orders, the numerical methods with different numerical accuracy with respect to the size of the time stepping can be obtained. We refer to [24] for the detail of the splitting methods, and [31] for the application of the splitting methods in the detonation simulations. The adaptive mesh methods also have been explored in depth on the simulations of the detonation phenomenon. The adaptive mesh methods optimize the distribution of the grid points according to some quantity such as the *a posteriori* error, so that the higher numerical accuracy can be obtained by using less grid points. This is an attractive technique for the detonation simulations since the solutions change dramatically only in a relatively small region, i.e., the detonation front and the following reaction zone, and the grid points of the mesh can be optimized by locating more points in those trouble regions. The author may refer to [5, 15, 34, 40] for the applications of the adaptive mesh methods in the detonation simulations. The author may also refer to [1, 9, 18, 30, 36, 37] for the successful applications of the adaptive mesh methods in other areas. Finally, with the development of the hardware, the parallel computing has been widely used to significantly accelerate the detonation simulations [29, 33].

Besides the efficiency, another issue on accurately resolving the dynamics of the detonation front is the outflow boundary conditions. In the ZND model, the detonation propagates in an infinitely long tube, with a constant velocity. To numerically study the wave propagation, a classical strategy is to introduce a moving frame with the same velocity, in which a stable detonation would be obtained, or the detonation front would