

Investigation of Riemann Solver with Internal Reconstruction (RSIR) for the Euler Equations

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Abstract. The Riemann solver with internal reconstruction (RSIR) of Carmouze et al. (2020) is investigated, revisited and improved for the Euler equations. In this reference, the RSIR approach has been developed to address the numerical resolution of the non-equilibrium two-phase flow model of Saurel et al. (2017). The main idea is to reconstruct two intermediate states from the knowledge of a simple and robust intercell state such as HLL, regardless the number of waves present in the Riemann problem. Such reconstruction improves significantly the accuracy of the HLL solution, preserves robustness and maintains stationary discontinuities. Consequently, when dealing with complex flow models, such as the aforementioned one, RSIR-type solvers are quite easy to derive compared to HLLC-type ones that may require a tedious analysis of the governing equations across the different waves. In the present contribution, the RSIR solver of Carmouze et al. (2020) is investigated, revisited and improved with the help of thermodynamic considerations, making a simple, accurate, robust and positive Riemann solver. It is also demonstrated that the RSIR solver is strictly equivalent to the HLLC solver of Toro et al. (1994) for the Euler equations when the Rankine-Hugoniot relations are used. In that sense, the RSIR approach recovers the HLLC solver in some limit as well as the HLL one in another limit and can be simplified at different levels when complex systems of equations are addressed. For the sake of clarity and simplicity, the derivations are performed in the context of the one-dimensional Euler equations. Comparisons and validations against the conventional HLLC solver and exact solutions are presented.

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1 Introduction

Derivation of an appropriate Riemann solver for complex flow models is a difficult task. Many waves may be present, some eigenvalues may have large multiplicity and some of the equations may be non-conservative. Examples of such models are the MHD equations (Balsara et al. (2012) [3]), compressible solid-fluid models (Gavrilyuk et al. (2008) [12]), non-equilibrium two-phase flow models such as the one of Saurel et al. (2017) [22], as well as the Godunov-Peshkov-Romenski (GPR) model (Peshkov et al. (2019) [19]). Also, when dealing with material interfaces, large density jumps may be present and volume fraction as well as density positivity is mandatory. Same requirement is needed for high speed flows, where vacuum conditions may appear. It seems that the most appropriate Riemann solver in these conditions is the HLLC solver of Toro et al. (1994) [28]. However, for specific models, its derivation may be non-trivial [25], [11].

In Carmouze et al. (2020) [4] an alternative is given and a new Riemann solver with internal reconstruction (RSIR) is designed. It relies on the following observation. It is usually quite easy to derive a single intermediate state solver, such as Rusanov (1961) [20] or HLL (Harten et al. (1983) [15]) even for complicated flow models. These solvers are very robust and positive but too dissipative for transport and stationary contact waves. Extensions of the HLL solver to include more wave information have been developed in Einfeldt et al. (1991) [9], Toro et al. (1994) [28], Linde (2002) [17] and Dumbser and Balsara (2016) [8].

In Carmouze et al. (2020) [4] the single intermediate solution is used to rebuild two intermediate states, thanks to an additional relation. These two intermediate states contain in most situations enough information to improve significantly accuracy and preserve robustness.

The contribution of Carmouze et al. (2020) [4] was mainly motivated by the numerical approximation of the non-equilibrium two-phase flow model of Saurel et al. (2017) [22] that involves a series of theoretical challenges as it is hyperbolic degenerate, presents non-conservative terms and exhibit non self-similar solutions. The complexity of the corresponding model prompted the authors to develop a solver based on internal reconstruction of intermediate states, computed from a simple and robust intercell state.

Thanks to the RSIR approach, stationary interfaces are maintained and numerical dissipation is reduced while circumventing the difficulties related to the construction of a HLLC-type solver. Chiapolino and Saurel (2020) [5] provide illustrative results of the RSIR solver in this two-phase flow context.

In Carmouze et al. (2020) [4], thermodynamic considerations have been introduced in the RSIR solver for the Euler equations and a similar treatment has been developed to address numerical resolution of Saurel et al. (2017) [22] model.

The underlying philosophy of this approach relies on the assumption that most of the physics is present in the two extreme waves and only one contact wave, that has to be identified. If the contact wave cannot be defined clearly, the method becomes irrelevant. But it seems that in most flow models such as the Euler equations and the