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A Computational Model for Simulation of Shallow Water Waves by Elastic Deformations in the Topography

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Abstract. We propose a coupled model to simulate shallow water waves induced by elastic deformations in the bed topography. The governing equations consist of the depth-averaged shallow water equations including friction terms for the water free-surface and the well-known second-order elastostatics formulation for the bed deformation. The perturbation on the free-surface is assumed to be caused by a sudden change in the bottom beds. At the interface between the water flow and the bed topography, transfer conditions are implemented. Here, the hydrostatic pressure and friction forces are considered for the elastostatic equations whereas bathymetric forces are accounted for in the shallow water equations. The focus in the present study is on the development of a simple and accurate representation of the interaction between water waves and bed deformations in order to simulate practical shallow water flows without relying on complex partial differential equations with free boundary conditions. The effects of location and magnitude of the deformation on the flow fields and free-surface waves are investigated in details. Numerical simulations are carried out for several test examples on shallow water waves induced by sudden changes in the bed. The proposed computational model has been found to be feasible and satisfactory.

AMS subject classifications: 74B10, 65M08, 35L53, 76B15, 74J40, 76B07

Key words: Shallow water equations, elastic deformation, sudden bed changes, finite element method, finite volume method, deformable beds.

1 Introduction

In this study the problem of shallow water flows under conditions of abrupt changes to the bathymetry is considered. The idea is to develop an accurate and efficient compu-

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tational approach to simulate such flows which pose a great challenge due to the differences in temporal and spatial scales. The free-surface flow which is commonly modeled by the shallow water equations coupled with the sudden changes of bathymetry is investigated numerically. It has been observed in nature that free-surface flows can sometimes be triggered by an abrupt or a sudden change in the bathymetry [1]. A clear example is the tsunamis or landslides that pour into a water body in oceans or lakes. The common practice in modeling such free-surface flows is to assume that the displacement of the free-surface is the same as the displacement of the bed, and it happens concurrently due to incompressibility of the water. Thus, as an initial condition one applies a static source together with a translation of the seabed deformation onto the free-surface flow. Such an approach was first presented for the field of modeling tsunamis in [6] and referred to as *the passive approach* in [8]. The validity of such an approach was reported in [9] among others. The passive approach neglects the so-called rupture velocity and the rise time of the fault/bathymetry change dynamics. Investigations have been undertaken to understand the rupture velocity and rise time in many engineering applications. For example, the study in [24] took into account the rise time, and the rupture velocity has also been accounted for in [12]. In [21], work has been undertaken in which a numerical integration of the time-dependent elasticity equations as well as time-dependent fluid equations was considered. Over the past years, the development of efficient and increasingly accurate numerical models of nonlinear shallow water equations over variable beds has been a continuous challenge in coastal engineering communities see [7, 20, 30] among others. Since these models are considered to be theoretically challenging and practically important, researchers are currently working on developing efficient and accurate computational tools to model shallow water waves by elastic deformations in the topography. This represents a great challenge due to the time and space scales for which the abrupt changes took places versus the evolution of the water free-surface.

The commonly used finite difference schemes in the numerical solution of the nonlinear shallow water equations are non-conservative leading to volume loss and energy dissipation as the wave steepness increases and the flow approaches discontinuities [31]. Recent advances in seismic inverse algorithms enable accurate descriptions of the rise time and rupture propagation over the source area. Furthermore, this model provides a time series of the vertical displacement and velocity that constitute the bottom boundary condition of a non-hydrostatic model for transfer of kinetic and potential energies to the water. One limitation of this method is the appearance of oscillations in the computational results when applied to more complex geometries [27]. The depth-integrated method was reformulated, and hence a non-hydrostatic model was derived in the spherical coordinate system for basin-wide waves propagation [32]. However, this method suffers from a drawback in that it cannot guarantee the conservation of momentum in the numerical scheme. Each of the aforementioned numerical modeling techniques has its own features and drawbacks when it is applied to the shallow water system. The finite difference scheme is considered to be easy to implement. However, the scheme is not accurate when dealing with complex geometries such as the characterization of