

Navier-Stokes Solvers for Incompressible Single- and Two-Phase Flows

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Abstract. The presented work is dedicated to the mathematical and numerical modeling of unsteady single- and two-phase flows using finite volume and penalty methods. Two classes of Navier-Stokes solvers are considered. Their accuracy and robustness are compared to identify their respective strengths and weaknesses. Exact (also referred to as monolithic) solvers such as the Augmented Lagrangian and the Fully Coupled methods address the saddle-point structure on the pressure-velocity couple of the discretized system by means of a penalization term or even directly, whereas approximate (segregated) solvers such as the Standard Projection method rely on operator splitting to break the problem down to decoupled systems. The objective is to compare all approaches in the context of two-phase flows at high viscosity and density ratios. To characterize the interface location, a volume-of-fluid (VOF) approach is used based on a Piecewise Linear Interface Construction (PLIC). Various 2D simulations are performed on single- and two-phase flows to characterize the behavior and performances of the various solvers.

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1 Introduction

The Navier-Stokes equations play a crucial role in several industrial and environmental applications that include weather forecast, combustion and propulsion, civil engineering, aerodynamics, and hydrodynamics. However, their resolution causes many problems and remains a challenging task for mathematicians as well as engineers. In the context of incompressible two-phase flows, the difficulties are two-fold: on the one hand, the

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velocity-pressure coupling induced by the incompressibility constraint, which must hold at every instant, gives rise to a saddle-point problem for which dedicated solvers must be designed. On the other hand, large density and viscosity ratios in the presence of large interface distortions yield ill-conditioned linear systems. Multiple strategies have been devised over the years to solve these ill-conditioned linear systems (the reader is referred to [7,9] for more information).

The devised solutions the pressure-velocity coupling challenge can be classified in two categories. On the one hand, segregated methods, introduced by Chorin [8] and Temam [36], approximate the original system using time splitting, thereby resulting in two decoupled equations: one to update the velocity field and the other the pressure field. This raises a number of issues, such as the appearance of numerical artifacts as a consequence of the splitting error, or the need for pressure boundary conditions that do not exist in the original problem. On the other hand, coupled methods solve both fields (velocity and pressure) simultaneously, hence preserving the consistency of the discretized system with the continuous equations (as opposed to segregated methods, which introduce a splitting error that is inconsistent with the continuous system). The system to be solved in coupled methods is known as a saddle point system on the pressure-velocity couple. This saddle point system can in turn be solved either directly (this strategy will be referred to as the Fully Coupled, or FC, method) or by means of a penalty method (Augmented Lagrangian, AL). The AL method was proposed initially by Fortin and Glowinski [14] primarily for single-phase Stokes flows. This approach was recently applied to two-phase flows by Vincent and coworkers [24, 38]. This is an iterative method, where each iteration requires the solution of the velocity equation modified to include an additional term that penalizes the incompressibility constraint. A subsequent update of the pressure field is performed explicitly. FC methods on the other hand are purely algebraic, using specific block preconditioners so as to improve the spectral properties of the original saddle point system. In recent years, significant efforts have been dedicated to this approach, and several techniques have been proposed in the context of single-phase flows, in the Stokes or steady regimes primarily, using finite element methods [11,26].

In the context of two-phase flow modeling at high density and viscosity ratios, several techniques have been investigated to tackle the induced stiffness. For projection methods (PR), Guermond and Salgado [18] have studied a new fractional time-step technique for variable density flows, which consist in extracting the density from the Poisson equation by penalizing the divergence of the velocity. The validity of this method is demonstrated for Rayleigh-Taylor instability at small density ratio. More recently, Dodd and Ferrante [10] have proposed a fast pressure-correction method for simulating incompressible two-phase flows with large density and viscosity ratios. The method is based on the splitting of the pressure gradient into two terms, one with a variable density which is explicitly treated and the other one with a constant density treated implicitly. The results obtained for a capillary wave show that this method is able to treat two-phase flow problems with density and viscosity ratios up to 10^5 . Nevertheless, the explicit treatment of the velocity prediction imposes a restriction on the time-step size to maintain numer-