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Continuous Data Assimilation with a Moving Cluster of Data Points for a Reaction Diffusion Equation: A Computational Study

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Abstract. Data assimilation is a technique for increasing the accuracy of simulations of solutions to partial differential equations by incorporating observable data into the solution as time evolves. Recently, a promising new algorithm for data assimilation based on feedback-control at the PDE level has been proposed in the pioneering work of Azouani, Olson, and Titi (2014). The standard version of this algorithm is based on measurement from data points that are fixed in space. In this work, we consider the scenario in which the data collection points move in space over time. We demonstrate computationally that, at least in the setting of the 1D Allen-Cahn reaction diffusion equation, the algorithm converges with significantly fewer measurement points, up to an order or magnitude in some cases. We also provide an application of the algorithm to a the estimation of a physical length scale in the case of a uniform static grid.

AMS subject classifications: 35K57, 35K40, 35K61, 37C50, 35Q93, 34D06

Key words: Continuous data assimilation, Allen-Cahn, reaction-diffusion, moving mesh, synchronization.

1 Introduction

Data assimilation refers to a wide class of techniques that aim to increase the accuracy of simulations by combining observational data together with physical models. Recently, a new method has emerged as a promising approach to data assimilation, known as Continuous Data Assimilation (CDA) or the Azouani-Olson-Titi (AOT) algorithm or after the authors who pioneered this idea in [4, 5]. The AOT approach introduces a feedback-control term at the level of the underlying partial differential equation (PDE) model by

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interpolating the observational data. In the standard implementation of the AOT algorithm, data is collected at fixed observation points on a static grid. Here, we examine the effect of allowing the observation points to move in time. As we demonstrate below, this can lead to order-of-magnitude improvements on rates of convergence and on the number of data points needed for convergence to occur. Indeed, such improvements were observed over a wide range of parameters, including those in which the equation is more computationally demanding; i.e., the regimes in which the nonlinear terms dominate the diffusion terms.

Classical data assimilation techniques are based on the Kalman Filter, a form of linear quadratic estimation. There are also variational methods collectively known as 3D/4D Var techniques. These methods are described in detail in several textbooks, including [15, 38, 41], and the references therein. The AOT algorithm is an entirely different approach that adds a feedback control term at the PDE level. A similar approach is followed in [11] in the context of stochastic differential equations. We note that, while the AOT method superficially appears similar to the nudging or Newtonian relaxation methods introduced in [3, 33]; however, the use of the interpolation in the AOT method is a major difference between the two methods, with crucial effects in terms of implementation, convergence rates, and the amount of measurement data required. For an overview of nudging methods, see, e.g., [39] and the references therein. A large amount of recent literature has built upon the AOT algorithm; see, e.g., [1, 2, 7–10, 12, 13, 21–31, 34–37, 40, 42, 44, 45, 47, 48]. Computational experiments on the AOT algorithm and its variants were carried out in the cases of the 2D Navier-Stokes equations [30], the 2D Bénard convection equations [2], and the 1D Kuramoto-Sivashinsky equations [40, 42].

In the present work, we focus on the particular case of a localized cluster of observation points moving at a uniform speed through the spatial domain. The scenario of moving observers may arise in realistic settings; e.g., a moving probe in an experiment, a vehicle or drone loaded with sensors as it crosses a crop, or a satellite sampling data as it orbits. Rather than study such complex settings however, we examine this technique in the context of a relatively simple (though still nonlinear) PDE, namely the one-dimensional Allen-Cahn reaction-diffusion equation. This PDE is also the one for which the AOT algorithm was first proposed and studied [5] (a more general algorithm was proposed and studied in [4] in the context of the 2D Navier-Stokes equations).

Moving observers have been used in conjunction with data assimilation in a wide variety of contexts, such as flow capturing [17, 18, 55], traffic state estimation [56], and autonomous robot formation [51]. However, moving observers can present a problem in classical data assimilation, since data are assimilated sequentially. To deal with this issue, one often assumes that all observations in a single "pass" of the observer through the spatial window of interest occur at the same time, or equivalently, that the dynamics of the observed system are sufficiently stationary [6, 49, 57]. This so-called stationarity assumption can result in biases which one can try to correct using still other techniques, as discussed in, e.g., [6]. For example, consider the following statement from [6].