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Multi-Resolution Method Based on Riemann Solvers for Detonation and Deflagration in High Dimension

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Abstract. In this paper, we propose accurate Riemann solvers for detonation and deflagration with sharp interface in high dimension. The standard finite volume scheme is used for each fluid away from material interface, the detonation and the deflagration interfaces are captured by the level set method, small cut cells are treated with a mixing procedure to get stable algorithm. By Riemann solver for the detonation and the deflagration, the interface fluxes are obtained. With the help of the adaptive multi-resolution algorithms, we extend the method to three dimension conveniently. Numerical examples in two or three-dimension are carried out to demonstrate the potential and robustness of the method.

AMS subject classifications: 76T10, 65M08

Key words: Compressible fluids, level set method, cut cell, Riemann solvers, adaptive multi-resolution, three dimension.

1 Introduction

Detonation is a multicomponent compressible fluid dynamical procedure, detonation wave is quite different from the usual shock wave, its front consists of a precursor shock wave that propagates into the unburned medium with a spike reaction zone behind the shock. The precursor shock compresses the unburned medium and increases its temperature, then detonation occurs behind the front, which can release energy that pushes the precursive shock to keep propagating forward continuously. To study this kind of fluid dynamic, the CJ and the ZND models are proposed in this field [30].

The CJ model developed by Chapman and Jouguet is considered as a limitation of instantaneous reaction [7]. In this theory, the detonation interface between the reactants

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and the products is just a single jump discontinuity, where the reaction finishes instantaneously. The ZND model is invented by Zeldovich, von Neumann and Doering independently, and it is a more complex model that can describe the subtle structure [20]. Unlike that in the CJ model, the reaction finishes in finite time, the detonation interface is followed by a chemical reaction zone.

According to the propagating velocity, the detonation can be classified as detonation and deflagration [5, 6], both are multi-fluid problems with reactants and products, and can be studied with the assumption that has sharp interface or reactive zone. In the case of sharp interface, with the help of the level set method, the detonation and deflagration are studied by Fedkiw et al. [18, 26], the ghost fluid method (GFM) [12] is implemented for contact discontinuities where the interface moves at the fluid velocity, the pressure and normal velocity of the ghost fluid are copied over from the real fluid, the entropy and tangential velocities are defined by extrapolation in normal direction. The GFM is extended to multicomponent interface [27, 31] where the interface velocity includes a regression rate due to the presence of chemical reactions, the numerical fluxes on the interface are defined as continuous variables in the GFM.

For three dimension case, adaptive and high order methods are needed accuracy and efficiency. High order methods has been studied for different problem [8, 10, 22, 28], both adaptive mesh refinement method (AMR) and multi-resolution method (MR) are essential for high dimensional problems [1,2,9,24]. Gao [14] used a parallel AMR algorithm to predict turbulent in combusting flows, Harten [16] proposed adaptive multi-resolution schemes for shock computation and come out with some numerical results. Bihari et al. [3, 4] applied the original multi-resolution schemes on the reactive Euler equations in one dimension, and performed it for non-reacting Euler equations in two dimension, Hejazialhosseini et al. [17] developed a new approach based on wavelet adaptive blocks with local time-stepping using a dynamic graded tree data structure, and this approach can easily perform in a highly parallel algorithms. Han et al. [15] perform the multi-resolution methods on non-reacting compressible flow and achieved high-order results.

In this paper, detonation and deflagration with sharp interface CJ models are considered, a standard finite volume scheme is employed in the single fluid region far away from the interface, the material interfaces are captured by level set method, because the scheme is in Euler frame, small cut cell will appear, in order to keep the stability of the numerical method, a mixing procedure is applied for the small cut cells as in Adams et al. [19]. The advantage of the mixing procedure is a separate step beyond the update of the conservative variable, it's convenient to extend the method to high dimension. For the detonation and deflagration cases, the transitions of mass, momentum and energy across the interface are obtained accurately by solving Riemann problems. For the efficiency, we proposed adaptive multi-resolution method for detonation and deflagration in three dimension.

This paper is organized as following: In Section 2, we describe the algorithm in detail, the governing equations and the equation of states based on the CJ model are given in Subsection 2.1, a conservative interface method is proposed in Subsection 2.2. The details