

## A Hybrid Immersed Boundary-Lattice Boltzmann Method for Simulation of Viscoelastic Fluid Flows Interaction with Complex Boundaries

M. H. Sedaghat<sup>1,\*</sup>, A. A. H. Bagheri<sup>2</sup>, M. M. Shahmardan<sup>2</sup>,  
M. Norouzi<sup>2</sup>, B. C. Khoo<sup>3</sup> and P. G. Jayathilake<sup>4</sup>

<sup>1</sup> School of Mechanical Engineering, Shiraz University, Shiraz, Iran.

<sup>2</sup> Department of Mechanical Engineering, Shahrood University of Technology, Shahrood, Iran.

<sup>3</sup> Department of Mechanical Engineering, National University of Singapore, Singapore 117575.

<sup>4</sup> Department of Oncology, University of Oxford, Oxford, OX3 7DQ, UK.

Received 8 September 2019; Accepted (in revised version) 21 February 2020

---

**Abstract.** In this study, a numerical technique based on the Lattice Boltzmann method is presented to model viscoelastic fluid interaction with complex boundaries which are commonly seen in biological systems and industrial practices. In order to accomplish numerical simulation of viscoelastic fluid flows, the Newtonian part of the momentum equations is solved by the Lattice Boltzmann Method (LBM) and the divergence of the elastic tensor, which is solved by the finite difference method, is added as a force term to the governing equations. The fluid-structure interaction forces are implemented through the Immersed Boundary Method (IBM). The numerical approach is validated for Newtonian and viscoelastic fluid flows in a straight channel, a four-roll mill geometry as well as flow over a stationary and rotating circular cylinder. Then, a numerical simulation of Oldroyd-B fluid flow around a confined elliptical cylinder with different aspect ratios is carried out for the first time. Finally, the present numerical approach is used to simulate a biological problem which is the muco-ciliary transport process of human respiratory system. The present numerical results are compared with appropriate analytical, numerical and experimental results obtained from the literature.

**AMS subject classifications:** 76-XX, 65-XX

**Key words:** Lattice Boltzmann method, immersed boundary method, viscoelastic fluid, complex boundaries, muco-ciliary transport.

---

\*Corresponding author. *Email addresses:* h.sedaghat@shirazu.ac.ir (M. H. Sedaghat), anaraki.mech@gmail.com (A. A. H. Bagheri), mmshahmardan@shahroodut.ac.ir (M. M. Shahmardan), mnorouzi@shahroodut.ac.ir (M. Norouzi), mpekbc@nus.edu.sg (B. C. Khoo), jayathilake.pahalagedara@oncology.ox.ac.uk (P. G. Jayathilake)

## 1 Introduction

Fixed mesh methods have been commonly employed for simulations of complex boundary problems to relieve computational cost associated with moving mesh methods. One of the robust fixed mesh methods is the Immersed Boundary Method (IBM) which was first proposed by Peskin [1] to model blood flow in human heart. In this method, Lagrangian points are located on the boundary of the object and Eulerian points are used to represent the fluid phase around the object. The idea of the IBM is the discretization of the momentum equations on the Eulerian points and the immersed boundary condition is depicted by adding a body force term into the momentum equations. Thereby, in this method the governing equations are decoupled from the computational mesh and as a result it can easily simulate not only flow past stationary complex geometries, but also flow interaction with moving objects. Early studies of the IBM also include simulating two-phase immiscible flows [2,3].

In previous studies, viscoelastic fluid flow in simple geometries have been solved using finite difference [4–6], finite element [7,8], spectral finite element [9] and finite volume [10–16] methods. De et al. [17] implemented finite volume and immersed boundary methods for simulating viscoelastic fluid flows around stationary cylinders. Krishnan et al. [18] used a similar method and presented a fully resolved simulation of particles moving in a viscoelastic fluid. Recently, the Lattice Boltzmann Method (LBM) has been introduced as a powerful method to simulate two phase flow [19], Newtonian as well as non-Newtonian fluid flows. The LBM is based on the mesoscopic kinetic equations for fluids which solve the discrete Boltzmann equation for the particle density distribution function [20]. This method has also been implemented for analysis of viscoelastic fluid flows and researchers have verified their approaches for benchmark problems including lid-driven cavity and Poiseuille flows [21–25]. Malaspinas et al. [26] proposed a new approach based on the LBM in order to simulate linear and non-linear viscoelastic fluids. In particular those described by the Oldroyd-B and FENE-P constitutive equations at low Reynolds numbers. Their model has some redundant terms which lack a clear physical meaning for the recovered stress tensor. Su et al. [27] modified the model of Malaspinas et al. [26] and proposed a novel numerical scheme for the simulation of viscoelastic fluid flows based on the LBM over a large range of Weissenberg numbers at low Reynolds numbers. Their results showed that viscoelastic fluid flow in two-dimensional channel flow is found to be in full accord with the analytical solution. Their discretization scheme for incorporating viscoelastic stress into the LBM is dependent on constitutive models, and is not general either. Zou et al. [28] using two open source Computational Fluid Dynamics (CFD) toolkits, namely OpenFOAM and OpenLB, proposed an integrated scheme based on the Lattice Boltzmann method and Finite Volume Method (FVM) for modeling incompressible polymer viscoelastic fluid flows. Their model has been critically validated using the Oldroyd-B model and linear PTT model under Poiseuille flow, Taylor-Green vortex flow and 4:1 abrupt planar contraction flow. In a similar investigation, Zou et al. [29] has modified their previous model by introducing an integrated Lattice