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A Cartesian-to-Curvilinear Coordinate Transformation in Modified Ghost Fluid Method for Compressible Multi-Material Flows

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Abstract. Modified ghost fluid method (MGFM) provides us an effective manner to simulate compressible multi-material flows. In most cases, the applications are limited in relatively simple geometries described by Cartesian grids. In this paper, the MGFM treatment with the level set (LS) technique is extended to curvilinear coordinate systems. The chain rule of differentiation (applicable to general curvilinear coordinates) and the orthogonal transformation (applicable to orthogonal curvilinear coordinates) are utilized to deduce the Cartesian-to-curvilinear coordinate transformation, respectively. The relationship between these two transformations for the extension of the LS/MGFM algorithm is analyzed in theory. It is shown that these two transformations are equivalent for orthogonal curvilinear grids. The extension of the LS/MGFM algorithm using the chain rule has a wider range of applications, as there is essentially no requirement for the orthogonality of the grids. Several challenging problems in two- or three-dimensions are utilized to validate the developed algorithm in curvilinear coordinates. The results indicate that this algorithm enables a simple and effective implementation for simulating interface evolutions, as in Cartesian coordinate systems. It has the potential to be applied in more complex computational domains.

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Key words: Compressible multi-material flows, curvilinear coordinate system, modified ghost fluid method, level set method, multi-material Riemann problem, coordinate transformation.

1 Introduction

The interaction of immiscible materials is of great relevance for many industrial and engineering applications. These applications include, but are not limited to, underwater

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explosions [1,2], high-speed flows with droplets [3,4], bubble dynamics [5,6], supersonic combustion [7, 8], inertial confined fusion [9] and impact and penetration [10, 11]. Efficient numerical simulation is often more crucial to understand those flow fields than experiments. A very challenging task in such problems is to properly handle the evolution of material interfaces. The difficulty becomes greater particularly when shocks and detonation waves impact on the interfaces.

Several numerical methods are available to represent interfaces and reflect the properties of two materials at the interfaces, each of which has its own characteristics and advantages. Some methods relax the sharp character of the material interfaces and thus regularize them over a small but finite region. Significant contributions can be found in [12–15] for γ-based models, where an additional transport equation in non-conservative form is solved to prevent oscillations, in [16, 17] for volume-of-fluid (VOF) methods, where the volume fraction is used to reconstruct interfaces, and so on. Various sharpening techniques are designed to prevent interfaces from being overly smeared or producing numerical instabilities, such as the tangent of hyperbola interface capturing (THINC) technique [18, 19], the anti-diffusivity technique [20–22] and the artificial compression technique [23, 24].

Some methods maintain the immiscible character and enable two materials sharing an interface to have totally different equations of state (EOS). Significant contributions or review papers can be found in [25, 26] for arbitrary Lagrangian Eulerian (ALE) methods, where the computational mesh near the interface deforms with the flow, in [27, 28] for front-tracking (FT) methods, where some markers are used to track interfaces, in [29, 30] for level set (LS) methods, where the zero level set of a signed-distance function is used to capture interfaces, and so on. Among these methods, the framework of ghost fluid method (GFM) [31, 32] can be utilized to define interface conditions in a straight-forward way and suppress undesired numerical oscillations.

By specially defining ghost fluid states in a GFM-based algorithm, the computation can be carried out as for a single-material manner. Fedkiw et al. [31] proposed the original GFM via using the local flow states or extrapolating from the real fluid to define ghost fluid states. In order to solve the problem with large contact discontinuities and strong pressure jumps at the material interface, Liu et al. [32] proposed the modified GFM (MGFM) via solving local multi-material Riemann problems and using the solutions to define ghost fluid states. The GFM-based algorithms assume that one can identify those cells directly adjacent to and enclosing the interface. One method using a representation of the interface (LS, FT, VOF, etc) is generally required. On account of the fact the LS method can deal with large interface deformations and topological changes rather easily, the LS/GFM is employed in most cases to simulate complex flow phenomena [33–39]. The FT/GFM [27, 40–42] and the VOF/GFM [43, 44] are also presented for various multi-material problems.

Most of the GFM-based algorithms are designed in Cartesian coordinate systems, which limits the applications to relatively simple geometries. Sarthou et al. [45] presented a second-order methodology to work with fictitious domains on curvilinear grids. The