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An Adaptive Finite Element PML Method for the Open Cavity Scattering Problems

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Abstract. Consider the electromagnetic scattering of a time-harmonic plane wave by an open cavity which is embedded in a perfectly electrically conducting infinite ground plane. This paper is concerned with the numerical solutions of the transverse electric and magnetic polarizations of the open cavity scattering problems. In each polarization, the scattering problem is reduced equivalently into a boundary value problem of the two-dimensional Helmholtz equation in a bounded domain by using the transparent boundary condition (TBC). An a posteriori estimate based adaptive finite element method with the perfectly matched layer (PML) technique is developed to solve the reduced problem. The estimate takes account both of the finite element approximation error and the PML truncation error, where the latter is shown to decay exponentially with respect to the PML medium parameter and the thickness of the PML layer. Numerical experiments are presented and compared with the adaptive finite element TBC method for both polarizations to illustrate the competitive behavior of the proposed method.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Electromagnetic cavity scattering, TM and TE polarizations, perfectly matched layer, adaptive finite element method, a posteriori error estimates.

1 Introduction

The phenomena of electromagnetic scattering by open cavities have attracted much attention due to the significant industrial and military applications in such areas as antenna synthesis and stealth design. The underlying scattering problems have been extensively

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studied by many researchers in the engineering and applied mathematics communities. We refer to the survey [20] and the references cited therein for a comprehensive account on analysis, computation, and optimal design of the cavity scattering problems.

In applications, one of particular interests is the radar cross section (RCS) analysis, which aims at how to mitigate or amplify a signal. The RCS is a quantity which measures the detectability of a target by radar system. Deliberate control in the form of enhancement or reduction of the RCS of a target is of high importance in the electromagnetic interference, especially in the aircraft detection and the stealth design. Since the problems are imposed in open domains and the solutions may have singularities, it presents challenging and significant mathematical and computational questions on precise modeling and accurate computing for the cavity scattering problems in order to successfully implement any desired control of the RCS. This paper concerns the numerical solutions of the open cavity scattering problems. We intend to develop an adaptive finite element method with the perfect matched layer (PML) technique to overcome the difficulties.

The PML technique was first proposed by Bérenger for solving the time-dependent Maxwell equations [7]. Due to its effectiveness, simplicity and flexibility, the PML technique is widely used in computational wave propagation [14, 15, 24, 25, 29]. It has been recognized as one of the most important and popular approaches for the domain truncation. Under the assumption that the exterior solution is composed of outgoing waves only, the basic idea of the PML technique is to surround the domain of interest with a layer of finite thickness of a special medium, which is designed to either slow down or attenuate all the waves propagating into the PML layer from inside of the computational domain. As either the PML parameter or the thickness of the PML layer tends to infinity, the exponential convergence error estimate was obtained in [17, 19] between the solution of the PML problem and the solution of the Helmholtz-type scattering problem. The convergence analysis of the PML problems for the three-dimensional electromagnetic scattering was studied in [6, 8, 9, 21].

In practice, if we use a very thick PML layer and a uniform finite element mesh, it requires very excessive grids points and hence involves more computational cost. In contrast, if we choose a thin PML layer, it is inevitable to have a rapid variation of the PML medium property, which renders a very fine mesh in order to reach the desired accuracy. On the other hand, the solutions of the open cavity scattering problem may have singularities due to the existence of corners of cavities or the discontinuity of the dielectric coefficient for the filling medium. These singularities slow down the speed of convergence if uniform mesh refinements are applied. The a posteriori error estimate based adaptive finite element method is an ideal tool to handle these issues.

A posteriori error estimators are computable quantities in terms of numerical solutions and data. They measure the error between the numerical solution and the exact solution without requiring any a priori information of the exact solution. A reliable a posteriori error estimator plays a crucial role in an adaptive procedure for mesh modification such as refinement or coarsening. Since the work of Babuška and Rheinboldt [4], the study of adaptive method based on a posteriori error estimator has become an active