Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2020-0079

Vol. **30**, No. 1, pp. 1-33
July 2021

A Polygonal Discontinuous Galerkin Formulation for Contact Mechanics in Fluid-Structure Interaction Problems

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Received 6 May 2020; Accepted (in revised version) 2 December 2020

Abstract. In this work, we propose a formulation based on the Polygonal Discontinuous Galerkin (PolyDG) method for contact mechanics that arises in fluid-structure interaction problems. In particular, we introduce a consistent penalization approach to treat the frictionless contact between immersed structures that undergo large displacements. The key feature of the method is that the contact condition can be naturally embedded in the PolyDG formulation, allowing to easily support polygonal/polyhedral meshes. The proposed approach introduced a fixed background mesh for the fluid problem overlapped by the structure grid that is able to move independently of the fluid one. To assess the validity of the proposed approach, we report the results of several numerical experiments obtained in the case of contact between flexible structures immersed in a fluid.

AMS subject classifications: 65M60, 75F10, 74M15

Key words: Polygonal Discontinuous Galerkin method, fluid-structure interaction, contact mechanics.

1 Introduction

The numerical modeling of the contact process in mechanical problems requires to introduce inequalities in the mathematical formulation, see e.g. [60, 85]. In particular, the contact could be modeled by means of two inequalities, one prescribing a kinematic non-penetration condition on the displacements, the other one prescribing a dynamic condition on the tractions; an additional equality ensuring the compatibility between these two

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conditions is added to the model, see e.g. [60, 85]. Moreover, depending on the application of interest, a friction law may be prescribed to model the tangential behaviour of the contact tractions [60]. From the numerical viewpoint, these conditions can be imposed via different approaches. In [38], a penalty method is employed to impose the kinematic non-penetration condition. In [35–37,39,40,69] and [25–27,76], the authors used a Nitsche-DG and augmented Lagrangian formulations, respectively, that guarantee the consistency of the numerical formulation. In [41, 44, 58, 59, 61, 75, 83, 84], a Lagrange multipliers technique is presented for prescribing both the kinematic and the dynamic non-penetration conditions.

Another important process involving many engineering applications is the interaction between a solid body and a fluid, see e.g. [19, 23, 46–48, 52, 53, 67, 72, 86, 87]. The inclusion of the contact model in a fluid-structure interaction (FSI) framework features additional challenges from the mathematical and numerical point of view, due to the need of modeling fluid slip at the fluid-structure interface, a condition that allows contact to take place. There are several applications that require the numerical approximation of FSI problems involving contact mechanics. For example, in the context of the modeling of the cardiovascular system, we mention the dynamics of the heart valves, see e.g. [12, 21, 54, 65, 74, 80, 81].

The physical process that takes place during the contact between two bodies immersed in a fluid has not been completely understood yet, though several recent works addressed the physical behaviour through experiments, see e.g. [18, 62, 71]. For this reason, the derivation of a suitable mathematical model that describes the contact in FSI with proper coupling conditions is not fully understood. From the theoretical viewpoint, several works show that the contact may happen only under specific conditions on the fluid and structure, or on the topology, see e.g. [13, 34, 50, 51, 57]. In particular, an important result [13, 34] shows that in the case of an incompressible fluid, for a smooth structure geometry, the standard *no-slip* kinematic coupling condition at the fluid-structure interface does not allow the contact between approaching bodies, since a thin fluid layer remains trapped between them. Thus, it is required to consider a *slip* condition to allow the contact.

For the numerical treatment of the contact, several approaches have been proposed in the literature. In [77], the authors proposed a fully-Eulerian approach to discretize the FSI problem by using no-slip conditions at the fluid-structure interface. This choice avoids the penetration between the structures, though it never allows their actual contact. In [49], the fully-Eulerian approach has been extended to the contact case with a penalization approach to prevent the penetration of the structures. In [78], the authors proposed a penalty approach to treat the contact problem in the framework of the Space-Time Finite Element method developed in [79]; in [20], a Fictitious Domain approach for the FSI problem with immersed thin structures is proposed and a penalty approach is employed to incorporate the non-penetration condition into the formulation. Lagrange multiplier approaches have been used as well to handle the contact conditions in FSI, see e.g. [12, 45, 68, 80].