Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2020-0059

Vol. **30**, No. 1, pp. 34-66
July 2021

A Novel Iterative Penalty Method to Enforce Boundary Conditions in Finite Volume POD-Galerkin Reduced Order Models for Fluid Dynamics Problems

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Received 26 March 2020; Accepted (in revised version) 28 September 2020

Abstract. A Finite-Volume based POD-Galerkin reduced order model is developed for fluid dynamics problems where the (time-dependent) boundary conditions are controlled using two different boundary control strategies: the lifting function method, whose aim is to obtain homogeneous basis functions for the reduced basis space and the penalty method where the boundary conditions are enforced in the reduced order model using a penalty factor. The penalty method is improved by using an iterative solver for the determination of the penalty factor rather than tuning the factor with a sensitivity analysis or numerical experimentation.

The boundary control methods are compared and tested for two cases: the classical lid driven cavity benchmark problem and a Y-junction flow case with two inlet channels and one outlet channel. The results show that the boundaries of the reduced order model can be controlled with the boundary control methods and the same order of accuracy is achieved for the velocity and pressure fields. Finally, the reduced order models are 270-308 times faster than the full order models for the lid driven cavity test case and 13-24 times for the Y-junction test case.

AMS subject classifications: 35Q25, 65N08, 76M12

Key words: Proper Orthogonal Decomposition, Navier–Stokes equations, Galerkin projection, penalty method, lifting function method, iterative method.

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1 Introduction

Complex fluid dynamics problems are generally solved using discretization methods such as Finite Difference, Finite Element, Finite Volume (FV) or spectral element methods. However, it is usually not feasible to use these methods for applications that require to be solved almost in real time, such as on-the-spot decision making, (design) optimization or control [1]. The high fidelity Computational Fluid Dynamics (CFD) tools, used for numerical simulations of the Navier–Stokes equations, are too computationally expensive for those purposes. This has motivated the development of reduced order modeling techniques. However, low degree-of-freedom models that are solely based on input-output data do not represent the physics of the underlying systems adequately and, moreover, may be sensitive to operating conditions [2].

Therefore, techniques, such as Reduced Basis (RB) methods, have been developed that retain the essential physics and dynamics of a high fidelity model that consists of discretized Partial Differential Equations (PDEs) describing the fluid problem [3, 4]. The basic principle of these reduced order methods is to project the (parametrized) PDEs onto a low dimensional space, called the reduced basis space, in order to construct a physics-based model that is reduced in size and, therefore, in computational cost [5–7].

Fluid flows can be controlled in several ways. As an example, the system configuration can be manipulated by modifying the physical properties. However, in this work the focus is on controlling boundary conditions (BC) that are essential for defining flow problems.

An example of a boundary control application from the nuclear field is the coupling of thermal-hydraulic system codes, i.e. transient simulations that are based on one-dimensional models of physical transport phenomena, with three-dimensional CFD codes [8, 9]. These type of system codes are, in general, based upon the solution of six balance equations for liquid and steam that are coupled with conduction heat transfer equations and that are supplemented by a suitable set of constitutive equations [10].

One of the main purposes of this coupling is to speedup the CFD calculations by only including the region of interest in the CFD model and the rest of the domain in the much faster system code. However, the gain in computational time of such a coupled model is still limited by the CFD part. To overcome this burden, the system codes can be coupled with reduced order models (ROM) of the high fidelity CFD codes. For transient problems, time-dependent boundary conditions of the ROM are then to be controlled based on the BCs obtained from the systems codes.

For industrial applications, the Finite Volume discretization method is widely used by commercial software and open-source codes, as the method is robust [11] and satisfies locally the conservation laws [12, 13].

By using a RB technique, the non-homogeneous BCs are, in general, no longer satisfied at the reduced order level. Furthermore, the BCs are not explicitly present in the ROM and therefore they cannot be controlled directly [14]. In literature [14–17], different approaches to control the ROM BCs can be found of which two common approaches