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Non-Intrusive Reduced Order Modeling of Convection Dominated Flows Using Artificial Neural Networks with Application to Rayleigh-Taylor Instability

Zhen Gao[1,*], Qi Liu[1], Jan S. Hesthaven[2], Bao-Shan Wang[1], Wai Sun Don[1] and Xiao Wen[3]

[1] *School of Mathematical Sciences, Ocean University of China, Qingdao, China.*

[2] *Mathematics Institute of Computational Science and Engineering, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.*

[3] *College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, China.*

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Abstract. A non-intrusive reduced order model (ROM) that combines a proper orthogonal decomposition (POD) and an artificial neural network (ANN) is primarily studied to investigate the applicability of the proposed ROM in recovering the solutions with shocks and strong gradients accurately and resolving fine-scale structures efficiently for hyperbolic conservation laws. Its accuracy is demonstrated by solving a high-dimensional parametrized ODE and the one-dimensional viscous Burgers' equation with a parameterized diffusion coefficient. The two-dimensional single-mode Rayleigh-Taylor instability (RTI), where the amplitude of the small perturbation and time are considered as free parameters, is also simulated. An adaptive sampling method in time during the linear regime of the RTI is designed to reduce the number of snapshots required for POD and the training of ANN. The extensive numerical results show that the ROM can achieve an acceptable accuracy with improved efficiency in comparison with the standard full order method.

AMS subject classifications: 35L65, 35Q35, 76N15

Key words: Rayleigh-Taylor instability, non-intrusive reduced basis method, proper orthogonal decomposition, artificial neural network, adaptive sampling method.

*Corresponding author. *Email addresses:* zhengao@ouc.edu.cn (Z. Gao), lq1697@stu.ouc.edu.cn (Q. Liu), jan.hesthaven@epfl.ch (J. S. Hesthaven), wbs@stu.ouc.edu.cn (B.-S. Wang), donwaisun@outlook.com (W. S. Don), xiaowen_ouc@163.com (X. Wen)

1 Introduction

In recent years, the Rayleigh-Taylor instability (RTI) [35, 40] phenomenon has gained increasing attention in many areas of scientific research, including interstellar medium and galaxy clusters [33], accretion onto the magnetospheres of neutron stars [43, 44], and filamentary structure on the Sun [26]. Due to the practical importance, a number of numerical methods have been developed to simulate the RTI, including the flux-corrected transport method [47], the level set method [29], the front tracking method [41], the smoothed particle hydrodynamics method [39], the boundary integral method [14], direct numerical simulations [13], large-eddy simulations [8], and the phase-field method [11, 38]. Besides, Zhang et al. [48–50] studied the weakly nonlinear incompressible single-mode RTI and the multi-mode RTI in spherical and planar geometries, and a two-dimensional thin shell model for the nonlinear RTI in spherical geometry is proposed by Zhao et al. [51].

Previous methods are prohibitively expensive when one seeks to reproduce the fine-scale structures of RTI, because they require repeatedly solving a high dimensional system of partial differential equations (PDEs) at a fine grid resolution. To seek alternatives to solve the full problem many times, one can regard the governing equations of RTI as a parameterized time-dependent PDE with parameterized initial conditions. The amplitude of the initial perturbation waves (A) and time (t) are considered as the parameters in two-dimensional parameter space. In this light, the reduced order modeling (ROM) methods [6, 27] have been taken into consideration due to its effectiveness in solving the parameterized problems. The main goal of the ROM is to replace the *full-order system* with a *reduced-order model* with a significantly smaller size, which results in a great decrease in the computational cost in CPU times and memory storages, with a controlled loss of accuracy [23].

Featuring an offline-online operational framework, the reduced basis method (RBM) [22, 34] is a powerful technique for the ROM methods of the parameterized problems. In general, RBM aims to approximate any member of the solution manifold with a low number of basis functions, the so-called reduced basis (RB), which are extracted from snapshots of the full-order solutions during the offline stage. The full-order system is projected onto the linear subspace spanned by the RB through a projection approach, such as Galerkin projection [17, 36]. Although the Galerkin procedure brings high accuracy, it also faces some challenges in computing the projection coefficients for complex nonlinear problems with a non-affine parametric dependence. To avoid this problem, a non-intrusive ROM method [9], in which the original system is only used to generate the snapshots and does not require the projection process [12], is proposed. Necessary techniques have been developed for the non-intrusive ROM methods, among which the proper orthogonal decomposition (POD) [30] is usually applied to extract the RB from the snapshots. In the last decade or so, a variety of methods based on the POD are introduced. In [2, 3], Audouze et al. proposed the non-intrusive ROM method based on the POD and radial basis functions (RBFs), where RBFs are used for approximating the solutions in the parameter domain. Guo and Hesthaven [20, 21] used the Gaussian process