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Isogeometric Analysis with Proper Orthogonal Decomposition for Elastodynamics

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Abstract. We consider reduced order modelling of elastodynamics with proper orthogonal decomposition and isogeometric analysis, a recent novel and promising discretization method for partial differential equations. The generalized-α method for transient problems is used for additional flexibility in controlling high frequency dissipation. We propose a fully discrete scheme for the elastic wave equation with isogeometric analysis for spatial discretization, generalized-α method for time discretization, and proper orthogonal decomposition for model order reduction. Numerical convergence and dispersion are shown in detail to show the feasibility of the method. A variety of numerical examples in both 2D and 3D are provided to show the effectiveness of our method.

AMS subject classifications: 35K20, 65M12, 65M15, 65M60

Key words: Isogeometric analysis, proper orthogonal decomposition, reduced order modelling, elastic wave, generalized-α method.

1 Introduction

Elasticity models are essential in applied mechanics and engineering. The temporal characteristics of elasticity have many applications in elastodynamics, e.g., building structure analysis [1], marine survey [2], geophysics [24, 25] and seismology [18, 26]. Numerical approaches for such problems include finite elements [27, 28], spectral elements [29, 30], mortar elements [31, 32], discontinuous Galerkin method [33, 34], etc. High accuracy can be achieved with large temporal or spatial resolution. The computational costs of discretization methods for such models with time, physical, geometry parameters are high

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when either time step or spatial size is relatively small. Of course, we can reduce the number of degrees of freedom by adaptive methods with locally refining the meshes. However, a posterior error estimates are not easy to be obtained for complex models with real applications. For real-time simulations and parameter optimization problems, rapid evaluation is essentially required but it is hard to achieve when the number of degrees of freedom of the discretized system is large. In such cases, we therefore resort to reduced order modeling [3].

Proper orthogonal decomposition (POD) is one popular reduced order modeling approach for problems of various interests in engineering such as turbulent flows [4, 39], weather forecasting and optimal control problems (see [3]). POD has been used widely in different fields with different names, e.g. principle component analysis in statistics and Karhunen-Loeve expansion in stochastic analysis. The combination of POD with Galerkin methods for time-dependent partial differential equations (PDEs) [5–7] extract pertinent information of the model system from the high-fidelity instances. The so-called *snapshots* are obtained for the construction of a low-dimensional basis together with the corresponding field information. A low-dimensional system which contains most information of the original system is built through the new basis. The error between the numerical solution from POD-Galerkin method and the exact solution consists of two parts, i.e., the error between snapshot and exact solution and the error between POD solution and snapshot [8, 9]. High-fidelity approximations obtained from a numerical discretization method for PDEs are crucial for obtaining accurate snapshots and POD-Galerkin solutions.

As an emerging method in recent years, Isogeometric analysis (IGA) [13] has been successfully applied to various fields including structural mechanics [12], fluid dynamics [10], acoustics [9, 37], electromagnetism [11], etc. IGA represents a generalization of the isoparametric finite element method and uses NURBS as basis functions, which possess advantages of exact geometry representation, high global regularity (up to C^{p-1}-continuous with p denoting the degree of the piecewise polynomials of the basis), and convenient integration into CAD software using NURBS design workflow. Complex multi-patch domains can be partitioned exactly in IGA. Moreover, efficient h-, p-, k- refinements and hierarchical scheme [40] can be used to increase accuracy and flexibility of numerical solutions.

The motivation of this paper stems from the fact that the accuracy of the reduced order solution obtained with a POD-Galerkin method first requires snapshots' high-fidelity at a reasonable expense of computational efforts. IGA can produce highly accurate numerical solutions for PDEs efficiently. The natural combination of IGA and POD has been successfully applied in parabolic problems [8, 42], convection dominated convection-diffusion-reaction [36], acoustic wave equation [9] and shape optimization problem [41]. The paper continues the previous works [8,9,36] and generalizes this model order reduction method for acoustic wave [9] to elastodynamics. We consider numerical dispersion of the method here. Dispersion analysis reveals how accurate the numerical schemes we proposed are with respect to different wave vectors. Dispersion analysis for vibrations