Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2020-0088

Vol. **30**, No. 2, pp. 448-485
August 2021

An Efficient Positivity-Preserving Finite Volume Scheme for the Nonequilibrium Three-Temperature Radiation Diffusion Equations on Polygonal Meshes

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Received 15 May 2020; Accepted (in revised version) 7 December 2020

Abstract. This paper develops an efficient positivity-preserving finite volume scheme for the two-dimensional nonequilibrium three-temperature radiation diffusion equations on general polygonal meshes. The scheme is formed as a predictor-corrector algorithm. The corrector phase obtains the cell-centered solutions on the primary mesh, while the predictor phase determines the cell-vertex solutions on the dual mesh independently. Moreover, the flux on the primary edge is approximated with a fixed stencil and the nonnegative cell-vertex solutions are not reconstructed. Theoretically, our scheme does not require any nonlinear iteration for the linear problems, and can call the fast nonlinear solver (e.g. Newton method) for the nonlinear problems. The positivity, existence and uniqueness of the cell-centered solutions obtained on the corrector phase are analyzed, and the scheme on quasi-uniform meshes is proved to be L^2- and H^1-stable under some assumptions. Numerical experiments demonstrate the accuracy, efficiency and positivity of the scheme on various distorted meshes.

AMS subject classifications: 65M08, 65M22

Key words: Radiation diffusion equations,positivity-preserving, high efficiency, stability, finite volume method.

1 Introduction

The nonequilibrium three-temperature (3-T) radiation diffusion equations are a kind of strongly nonlinear partial differential equations, and are widely used to describe the energy evolution and exchanges among the electrons, ions and photons in a multi-material system. They simulate the radiation transport and arise in a wide range of applications

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such as the astrophysics and the inertial confinement fusion (ICF), see e.g. [6,7,12,15], if the radiation field is not in thermodynamic equilibrium with the material and the material itself is not in equilibrium. Developing the high-order accurate and efficient numerical scheme for the nonequilibrium 3-T radiation diffusion equations is very challenging and has drawn many researchers' attention.

In recent years, some numerical methods have been developed for solving the 3-T radiation diffusion equations, such as the finite volume (FV) methods in [13,21,43], the finite difference methods in [2,11] and the finite element method in [25]. In [21], a fully implicit finite volume scheme combined with parallel adaptive multigrid method was studied in the framework of UG. In [13], a symmetric finite volume element method (SFVE) was designed with a preconditioning technique, a mesh adaptation algorithm and a two-grid procedure. In [43], two substructuring nonoverlapping domain decomposition preconditioners were employed to solve the SFVE discretization of 2D 3-T radiation diffusion equations with strongly discontinuous coefficients. In [2,11], the Newton-Krylov method was used to solve the finite difference schemes for 2D 3-T radiation equations. In [25], the freezing coefficient method was adopted to linearize the nonlinear equations and then the resulting equations were solved by the Raviart-Thomas mixed finite element method. Unfortunately, those schemes only work on the rectangular or triangular meshes.

In many applications such as the radiation hydrodynamics solved by the Lagrangian method, the meshes become usually distorted and concave with or without hanging nodes due to the complex fluid flow. Developing an efficient numerical scheme for the 3-T radiation diffusion equations on arbitrary polygonal meshes is one of the significant requirements. Recently, some schemes have been proposed on general polygonal meshes. In [41], a Lions domain decomposition algorithm based on a cell functional minimization scheme was studied on non-matching multi-block grids for the nonlinear radiation diffusion equations. In [5], a set of numerical schemes were developed for the 3-T radiation diffusion equations in systems involving multi-materials. In [32], a monotone cell-centered scheme for diffusion equation was used for the 3-T radiation diffusion equations on convex polygons and the interpolation algorithms suggested in [9] was adopted to evaluate the cell-vertex unknowns.

In many situations, another significant requirement of the numerical schemes for the 3-T radiation diffusion equations is that the discrete solution should be nonnegative. Prior non-negative formulations for single diffusion problems were broadly studied over the years. A typical formulation is constrained optimization-based finite element method for transient diffusion equations, see standard linear finite element method [19], mixed finite element methods [24], the classical Galerkin formulation [22,23]. It is not suitable for polygonal meshes. Le Potier [29] proposed a monotone cell-centered finite volume scheme for the steady-state diffusion equations on triangular meshes. Since then, this approach have been developed to obtain numerous positivity-preserving finite volume schemes on general grids, see [4,10,16,17,33,44,45] and the references therein. This paper is concerned with the finite volume schemes. There exist some positivity-preserving