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A Hermite WENO Method with Modified Ghost Fluid Method for Compressible Two-Medium Flow Problems

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Abstract. In this paper, we develop a novel approach by combining a new robust finite difference Hermite weighted essentially non-oscillatory (HWENO) method [51] with the modified ghost fluid method (MGFM) [25] to simulate the compressible two-medium flow problems. The main idea is that we first use the technique of the MGFM to transform a two-medium flow problem to two single-medium cases by defining the ghost fluids status based on the predicted interface status. Then the efficient and robust HWENO finite difference method is applied for solving the single-medium flow cases. By using immediate neighbor information to deal with both the solution and its derivatives, the fifth order finite difference HWENO scheme adopted in this paper is more compact and has higher resolution than the classical fifth order finite difference WENO scheme of Jiang and Shu [14]. Furthermore, by combining the HWENO scheme with the MGFM to simulate the two-medium flow problems, less ghost point information is needed than that in using the classical WENO scheme in order to obtain the same numerical accuracy. Various one-dimensional and two-dimensional two-medium flow problems are solved to illustrate the good performances of the proposed method.

AMS subject classifications: 65M60, 35L65

Key words: Hermite WENO scheme, two-medium flow problems, modified ghost fluid method, Hermite interpolation.

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1 Introduction

In this paper, a robust finite difference Hermite weighted essentially non-oscillatory (HWENO) method recently developed in [51] is combined with the modified ghost fluid method (MGFM) [25] to construct a novel approach in simulating the compressible two-medium flow problems. The compressible two-medium flow problems have different equation of state (EOS) across the material interface. This property causes challenge in designing efficient high order accuracy schemes since numerical oscillations or inaccuracies may easily appear in simulation results. In the literatures, there are two major ways to solve the compressible two-medium flow problems. One is the front capturing method, in which the high resolution methods with numerical diffusion or viscosity are used. The major advantages of the front capturing method are its ability to deal with large deformation problems and extension to high dimension easily, but the numerical inaccuracies or oscillations may still appear nearby the interface. Hence, there are various techniques introduced by e.g. Larrouturou [17], Karni [16], Abgrall et al. [1,2], Shyue et al. [39], Saurel et al. [36] and Chen et al. [7] to resolve this issue. The other one is the front tracking method, where the discontinuities between the two-medium flows are treated as internal moving interfaces. The method works very well across multi-material interfaces, however it could lead to the entanglement of the Lagrangian meshes, and the extension to high dimension is more difficult than the front capturing method. Classical works on the front tracking approach includes e.g. volume of fluid (VOF) method [12], level set method [40] and other front tracking methods [9,42].

To combine the advantages of the front capturing and tracking methods, Fedkiw et al. [8] constructed a new numerical method named as the ghost fluid method (GFM). In GFM, a level set function is used to track the interface in Eulerian schemes, which makes the interface "invisible". The pressure and velocity at the ghost fluid nodes near the material interface are defined as the local real values, while the density is obtained by isobaric fixing. The method transforms a two-medium flow problem to two single-medium flow problems via defining the ghost fluid status. Then, various state-of-the-art schemes for the single-medium flow problems can be applied straightforwardly. The GFM offers a flexible approach to solve the two-medium flow problems, furthermore, the extension of the method to solve high dimensional problems is fairly easy and straightforward. In complicated problems that a strong shock wave impacts on the interface, numerical inaccuracies often appear at the interface, if the wave interaction and material properties on both sides of the interface are not taken into account in the GFM. To solve this issue, Liu et al. [25] developed a modified ghost fluid method (MGFM), in which they defined the ghost fluid values using the predicted interfacial status obtained by solving a multi-material Riemann problem exactly or approximately. By taking the interaction of shock waves with the interface into account, the MGFM combines the advantages of the GFM [8] and the implicit characteristic methods [23,24]. Later on, the interface interaction ghost fluid method (IGFM) [13], the real ghost fluid method (RGFM) [43] and the practical ghost fluid method (PGFM) [44] were developed following the idea of the Riemann