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Linear Stability Analysis and Gas Kinetic Scheme (GKS) Simulations of Instabilities in Compressible Plane Poiseuille Flow

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Abstract. The fundamental nature of flow instability in wall bounded flows changes with Mach number. The objectives of this study are two-fold, (i) compute the instability modes in high Mach number Poiseuille flows using linear stability analysis (LSA) and, (ii) perform direct numerical simulations (DNS) of the instability development using a solver based on gas kinetic method (GKM) for the purpose of code validation by comparison against LSA results. The LSA and DNS are performed for the case of Poiseuille flow over a range of Mach numbers – from moderately supersonic to hypersonic speeds. First, LSA is employed to identify the most unstable mode over the range of Mach numbers. We then perform two sets of GKM-DNS to corroborate the LSA results over the Mach number range. In the first set of simulations, the background field is initially perturbed with the most unstable mode identified by LSA and the evolution is monitored. It is shown that GKM-DNS accurately captures the exponential growth in kinetic energy for all Mach numbers. The second set of GKM-DNS simulations is performed by superposing the background pressure field with random initial perturbations. After an initial transient period, the modes predicted by LSA dominate the DNS flow field evolution. The wave-vector and mode shapes of the dominant instability are well replicated by GKM-DNS at each Mach number. These insights in the linear regime of high speed Poiseuille flow and validation of GKM are important for understanding and simulating wall bounded flows.

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Key words: Gas kinetic method, compressible linear stability theory, hypersonic flows.

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1 Introduction

The transition from laminar to turbulent flows in boundary layers at hypersonic speeds has been of keen interest to researchers for more than six decades. Identifying dominant linear instabilities is a key component in the understanding of laminar to turbulent transition process in wall bounded flows (Zhang et al. [1], Zhu et al. [2] and Zhu et al. [3]). Mack [4] performed an analysis of the instabilities as a function of Mach number in boundary layer flows. At low Mach numbers, Tollmien-Schlichting (T-S) waves also known as first mode are most unstable according to linear stability analysis. On the contrary, in high Mach number flows, multiple instability modes (second and higher) belonging to the family of trapped acoustic waves can coexist along with the first mode. It is well known that while the first mode instability is most unstable in the supersonic regime, the second or Mack mode dominates in the hypersonic regime for insulated plates [5]. For cooled flat plate boundary layers, the shift from first mode to Mack mode can even occur at lower Mach numbers. Contrary to high speed boundary layer flows, confined flows such as plane Couette flow exhibit two families of unstable acoustic modes (Duck et al. [6] and Hu and Zhong [7]). Much like the Mack mode, these two families of acoustic modes, modes I and II are created by sustained acoustic reflections between a wall and a relative sonic line when the relative Mach number in the local region is supersonic. Hu and Zhong [7] showed that Mode II originating due to trapped acoustic waves between the upper wall and sonic line is the dominant instability for all Mach numbers.

Similar to plane Couette flow, Poiseuille flow is an archetypal wall-bounded flow. In the incompressible regime, Poiseuille or channel flow has been extensively used to investigate stability, transition and turbulence phenomena (Orszag [8], Rempfer [9], Sandham and Kleiser [10]). At these low speeds, channel flow is itself of practical importance and, further, many of the flow mechanisms are similar to those in boundary layers. Therefore, there have been many experiments and numerical simulations of Poiseuille and channel flows. Contrary to low speed flows, high speed channel flows can neither be realized easily in a laboratory experiment, nor so it is of direct practical importance. Nonetheless, it exhibits some of the key features of high speed boundary layers. Thus, analyzing the stability characteristics of high speed Poiseuille flow is still of academic value. Further, and perhaps more importantly, Poiseuille flow can be used for validating new computational approaches by comparison against analytical results.

Current computational methods almost exclusively use compressible Navier-Stokes solvers for performing high speed flow simulations. In many cases, the accuracy of the solution requires careful alignment of the grid with the bow shock (Sivasubramanian and Fasel [11,12]) or wavenumber filtering to control numerical instabilities (Franko and Lele [13]). Various modifications are required to account for high-speed and high-enthalpy effects. For hypersonic flow conditions, using numerical schemes based on the more fundamental Boltzmann equation can be potentially advantageous. Gas-kinetic simulations are inherently better suited for simulating non-equilibrium and rarefied effects present in the hypersonic regime (Xu [14], Xu et al. [15], Ohwada and Xu [16]).