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Phase-Field-Based Axisymmetric Lattice Boltzmann Method for Two-Phase Electro-Hydrodynamic Flows

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Abstract. In this work, a novel and simple phase-field-based lattice Boltzmann (LB) method is proposed for the axisymmetric two-phase electric-hydrodynamic flows. The present LB method is composed of three LB models, which are used to solve the axisymmetric Allen-Cahn equation for the phase field, the axisymmetric Poisson equation for the electric potential, and the axisymmetric Navier-Stokes equations for the flow field. Compared with the previous LB models for the axisymmetric Poisson equation, which can be viewed as the solvers to the convection-diffusion equation, the present model is a genuine solver to the axisymmetric Poisson equation. To test the capacity of the LB method, the deformation of a single leaky or perfect dielectric drop under a uniform electric field is considered, and the effects of electric strength, conductivity ratio, and permittivity ratio are investigated in detail. It is found that the present numerical results are in good agreement with some available theoretical, numerical and/or experimental data.

AMS subject classifications: 76T10, 76M28, 76W05

Key words: Phase field, axisymmetric lattice Boltzmann method, two-phase electro-hydrodynamic flows.

1 Introduction

Electrohydrodynamics (EHD) is the study of the charged fluid dynamics. It is an interdisciplinary field emerging with the development of hydrodynamics and electrodynamics, and has also been of significance in many industry applications, for instance, the emulsion breaking, the ink-jet printing and the EHD pump, to name but a few [1–4].

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Two-phase EHD flows, especially the deformed drops subjected to a uniform electric field, have received increasing attention due to their importance in Engineering [5]. O'Konski and Thacher [6] first conducted a theoretical analysis on a perfect dielectric (without any ions or free-electrons) drop where only the polarization effect in the normal direction on the interface is considered, and a prolate drop deformation in the steady state is observed. Later, Taylor [7] studied the leaky dielectric liquids that carry some free charges. Under the effect of the electric properties, the surface charge would be accumulated at the interface. Owing to the inclusions of the surface charge and its contribution on tangential stresses on the interface, the leaky dielectric model [7, 8] can predict the prolate or oblate pattern of a weak conductivity drop with a small deformation. Up to now, the Taylor's model has been served as a classic benchmark to validate numerical methods for the two-phase EHD flows. On the other hand, when both liquids are perfect dielectrics, Shewood presented another benchmark problem of a small drop deformation [9]. Although there are some further theoretical results [10–12] which can be consider as the extensions to the pioneering works [7, 8], they are still limited to the case of a small drop deformation.

To gain a deeper insight into the two-phase EHD flows and interface dynamics, some different approaches have been developed, including the boundary-integral method [9, 13], the volume-of-fluid method [14, 15], the front-tracking method [16–18], the level-set method [19, 20], and the phase-field method [21–23]. Among these methods, the phase-field method has made a great progress owing to its capacity in capturing the complex topological change of interface. In this method, the interface between different phases is depicted by the conservative Cahn-Hilliard (C-H) [24, 25] or Allen-Cahn (A-C) [26] equation, the electric potential is described by the Poisson equation, and the fluid flow is governed by the Navier-Stokes equations. In this work, we will focus on the lattice Boltzmann (LB) method [27–30] which is different from the traditional methods for the two-phase EHD flows considered in some previous works [9, 13–23]. Due to the mesoscopic nature, the LB method has some distinct advantages, including easy implementation of the complex boundary conditions, parallelism in algorithm, and simplicity in coding. Thus it has become a powerful numerical tool for complex flows involving the interfacial dynamics [27–30]. Actually, in the past years, the LB method has also been used to study EHD flows [31–41]. Zhang and Kwok first used the LB method to study two-dimensional (2D) leaky dielectric drop by coupling the pseudo-potential model for multiphase interaction [31]. Later, the pseudo-potential based LB method has also been extended to investigate some more complex EHD flows, including the dynamics of the electrostriction bubbles [32], the instability of the EHD drop [33], the EHD drop with a large density ratio [34], the dynamics of the falling EHD drop [35], the EHD drop formation in microchannels [36], the interaction of the EHD drop pairs [37] and the electrified jet [38]. In addition to the pseudo-potential based LB model, the color-gradient LB model [39, 40] and phase-field based LB model [41] are also developed for the two-phase EHD flows. However, the above LB models are only proposed in Cartesian coordinate system, and have some limitations for axisymmetric EHD flows. This is because