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A Non-Ersatz Material Approach for the Topology Optimization of Elastic Structures Based on Piecewise Constant Level Set Method

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Abstract. The topology optimization of a linearized elasticity system with the area (volume) constraint is investigated. A non-ersatz material approach is proposed. By introducing a fixed background domain, the linearized elasticity system is extended into the background domain by a characteristic function. The piecewise constant level set (PCLS) method is applied to represent the original material region and the void region. A quadratic function of PCLS function is proposed to replace the characteristic function. The functional derivative of the objective functional with respect to PCLS function is derived, which is zero in the void region and nonzero in the original material region. A penalty gradient algorithm is proposed. Four numerical experiments of $2D$ and $3D$ elastic structures with different boundary conditions are presented, illustrating the validity of the proposed algorithm.

AMS subject classifications: 49R05, 47A75, 65F18, 65N25

Key words: Non-ersatz material approach, piecewise constant level set method, linearized elasticity system, sensitivity analysis, penalty gradient algorithm.

1 Introduction

Structural topology optimization is an effective tool to achieve maximum efficiency in a structure design [7]. Four major methods, including homogenization method [4], evolutionary structural optimization method (ESO) [25,31], solid isotropic material with penalization (SIMP) method [3] and level set method (LSM) [19], have been highly developed in the past decades [26].

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Level set method (LSM), proposed by Osher and Santosa [13], could express the geometry of a domain simply and efficiently by a level set function (a signed distance function). Sethian and Wiegmann first introduce LSM into the structural shape and topology optimization [16]. Osher and Santosa investigate the maximization of the smallest eigenvalue and the gap of first two eigenvalues of the eigenvalue problem [13]. Wang et al. study the structural topology optimization by applying the LSM [19]. Allaire et al. use LSM to solve the minimization of compliance and the minimization of the squared error between the computed displacement and the target displacement [2]. They also extend the LSM to deal with the vibration problems with the objective functions such as eigenfrequencies and multiple loads [1]. For the reduction of structure vibration with an excitation frequency or a frequency range, the minimization of the frequency response at the specified points or surfaces on the structure is investigated [17]. Xia et al. [23] apply LSM to deal with the maximization of the simple and multiple smallest eigenvalue of a vibration equation. Theillard et al. [18] propose a second-order numerical scheme to solve the linearized elasticity equations in irregular domains. Based on LSM, maximization of the structure stiffness is investigated. To optimize a structure with multiple-type boundaries, LSM is proposed to represent and propagate different boundary segments [21,24]. Polajnar et al. propose a novel stress-related objective function with the global stress-deviation measure [14]. To avoid the emergence of isolated region and the localized mode, Li et al. propose a mode recognition technique based on LSM [9].

The piecewise constant level set (PCLS) method, proposed by Christiansen and Tai [5], is an alternative LSM. Different domains are presented by different constants. Thus, the re-initialization process of the level set function is unnecessary. Instead, a simple constraint is added to keep PCLS function as a piecewise constant level set function. Furthermore, compared to the traditional LSM, PCLS method could use more general initialization. It is unnecessary to dig holes as many as possible at the beginning. In the process of iteration, holes can be created and eliminated automatically.

The PCLS method is originally proposed for image segmentation [10], and obtains satisfactory segmentation results. Wei and Wang introduce PCLS to solve a structural shape and topology optimization problem [20], where the strain energy is used as the objective functional. For the same objective functional, the variational binary level set method is applied by Dai et al. to solve the [6]. PCLS method is extended into to the boundary control of eigenvalue optimization problem by Zhang et al. [29] and elastic contact problems by Myslinski [12]. Zhang and Chen investigate the maximization of the smallest eigenfrequency of the linearized elasticity system and topology optimization of damping layer under harmonic excitations based on PCLS method [27,28]. PCLS method is extended into the damage identification of continuum structures based on natural frequencies [30].

For the structural optimization problem, the ersatz material approach is applied before using the LSM and PCLS method [1,20]. That is, the void region is filled by weak material and the original elasticity system is extended into a large and fixed background domain. The structural optimization of one material is changed into two materials (weak and original materials). This approach provides a good approximation to the original