

Consistent Forcing Scheme in the Simplified Lattice Boltzmann Method for Incompressible Flows

Yuan Gao¹, Liuming Yang¹, Yang Yu¹, Guoxiang Hou^{1,*} and Zhongbao Hou²

¹ School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China.

² Department of Medical Biotechnology and Laboratory Science, Chang Gung University, Taoyuan, 33302, Taiwan, China.

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Abstract. Considering the fact that the lattice discrete effects are neglected while introducing a body force into the simplified lattice Boltzmann method (SLBM), we propose a consistent forcing scheme in SLBM for incompressible flows with external forces. The lattice discrete effects are considered at the level of distribution functions in the present forcing scheme. Consequently, it is more accurate compared with the original forcing scheme used in SLBM. Through Taylor series expansion and Chapman-Enskog (CE) expansion analysis, the present forcing scheme can be proven to recover the macroscopic Navier-Stokes (N-S) equations. Then, the macroscopic equations are resolved through a fractional step technique. Furthermore, the material derivative term is discretized by the central difference method. To verify the results of the present scheme, we simulate with multiple forms of external force interactions including the space- and time-dependent body forces. Hence, the present forcing scheme overcomes the disadvantages of the original forcing scheme and the body force can be accurately imposed in the present scheme even when a coarse mesh is applied while the original scheme fails. Excellent agreements between the analytical solutions and our numerical results can be observed.

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Key words: Simplified lattice Boltzmann method, force term implementation, non-uniform body force, Chapman-Enskog analysis, the discrete effect.

1 Introduction

Over the past few decades, the lattice Boltzmann method (LBM) [1–6] has been developed into a popular and promising numerical scheme in the field of computational fluid

*Corresponding author. *Email addresses:* houguoxiang@163.com (G. Hou), hustgaoyuan@outlook.com (Y. Gao), yanglm_hust@foxmail.com (L. Yang), yyyfx@hotmail.com (Y. Yu), Bao835711082@gmail.com (Z. Hou)

dynamics (CFD) [7–12]. Unlike conventional CFD methods based on direct discretization of the NavierStokes (N-S) equations, LBM solves a discrete kinetic equation at the mesoscopic scale. In LBM, a fluid is modeled as pseudo particles and a linear evolution of distribution functions is reflected locally, which makes it easy to study the interactions among the fluids and full parallelism [13–16]. However, some drawbacks also exist in the original LBM, which include the difficulty to be applied on non-uniform grids or for complex geometry, more cost in virtual memories, and the instability at high Reynolds number [17–22].

A methodology which is based on the lattice Boltzmann equation (LBE) with BGK collision operator (LBGK) named simplified lattice Boltzmann method (SLBM) [23–26] is newly developed to address the shortcomings of the LBM. With the fractional step technique [27], the governing equations recovered from LBGK by using the Chapman-Enskog (C-E) multi-scale expansion can be resolved in SLBM. Moreover, SLBM reconstructs solutions to the equations above in the LBE frame with the second-order accuracy in space. In practical computations, the equilibrium distribution function f_{α}^{eq} which can be computed from the macroscopic properties is updated and the difference of two equilibrium distribution functions at two different locations and time levels is adopted to evaluate the non-equilibrium distribution function f_{α}^{neq} . In this way, SLBM avoids the necessity of storing the density distribution function f_{α} along all lattice velocity directions at every grid node. Such treatments bring several nice features to the SLBM: (1) it saves a lot of virtual memories; (2) the physical boundary conditions can be directly implemented. Additionally, SLBM has shown its promise in various flow problems like the axisymmetric flows [28], the thermal flows [29, 30], the non-Newtonian flows [31], and the multiphase flows [32, 33].

For the continuous Boltzmann equation, the contribution of a body force $F = \rho a$, where a is the force acceleration, is reflected in the change to the particle velocity by a forcing term $a \cdot \nabla_{\xi} f(x, \xi, t)$ (where $f(x, \xi, t)$ is the distribution function in the continuous Boltzmann equation, x is the position of the particle, ξ represents its velocity and t is the time). But for discrete Boltzmann systems such as LBM, and SLBM, where the velocity of the particles is discretized, the so-called lattice discrete effects must be minimized to derive correct results for fluid flows [34, 35]. Hence, the external forcing terms call for some special treatment and is of importance for the SLBM since forces play such an important role in fluid dynamics such as the multiphase flows, the thermal flows, and the fluid-structure interactions. The discrete effects have attracted considerable attention in the LBM community and several forcing schemes have been proposed [36–42] to solve the incompressible NavierStokes equations with appropriate accuracy. For example, He et al. [36] derived from the forcing term $a \cdot \nabla_{\xi} f(x, \xi, t)$ in the continuous Boltzmann equation. It used the equilibrium distribution function $f^{eq}(x, \xi, t)$ to approximate $f(x, \xi, t)$, and developed an approximate expression for the forcing term. Luo et al. pointed out that although the external forcing term $a \cdot \nabla_{\xi} f(x, \xi, t)$ in the continuous Boltzmann equation cannot be directly discretized, the velocity moment of this term can be calculated. Following this thought, a representation of the forcing term was proposed [37]. Later, Guo et