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A Neural Network Approach to Sampling Based Learning Control for Quantum System with Uncertainty

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Abstract. Robust control design for quantum systems with uncertainty is a key task for developing practical quantum technology. In this paper, we apply neural networks to learn the control of a quantum system with uncertainty. By exploiting the auto differentiation function developed for neural network models, our method avoids the manual computation of the gradient of the cost function as required in traditional methods. We implement our method using two algorithms. One uses neural networks to learn both the states and the controls and one uses neural networks to learn only the controls but solve the states by finite difference methods. Both algorithms incorporate the sampling-based learning process into the training of the networks. The performance of the algorithms is evaluated on a practical numerical example, followed by a detailed discussion about the advantage and trade-offs between our method and the other numerical schemes.

AMS subject classifications: 49N05, 68Q12, 93E03

Key words: Optimal control, quantum computing, uncertainty quantification, deep neural network.

1 Introduction

The control of quantum systems plays an important role in developing practical quantum technologies such as quantum sensing, optical spectroscopy, magnetic resonance imaging, etc. [1–3]. The main problem is transforming an initial quantum state to a final target

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state using electromagnetic control fields [4], which is done by maximizing the fidelity of the quantum system. While quantum optimal control is well studied theoretically if the gradient of the cost can be calculated analytically, robustness becomes a key issue in the practical modeling and control of real quantum systems due to the existence of uncertainty, commonly due to imperfect knowledge of the system or to fluctuations in experimental parameters and control error. [5, 6]. Moreover, the uncertainty in a system also introduces sub-optimal local minimum to the control landscape [7]. Therefore a robust control design is highly desirable in the analysis of the quantum system in practice. A theoretical study of control noises in a quantum system can be found in [8]. A list of robust quantum control methods includes sliding mode control [9], adiabatic passage [10], single-shot shape pulse [11], sequential convex programming [12], composite pulses [13], noise filtering [14], smooth optimal control [15] and sampling-based learning control [16].

Neural network (deep learning) has been one of the fastest-growing fields since 2012 and quickly expands to a wide area of research. Recently multiple efforts have been made to demonstrate the underlying connection between discrete dynamical systems and several state-of-the-art deep network architectures [17, 18]. Since then many have been exploring the applications of neural network technics in solving differential equation [19–21], partial differential equation [22–26], stochastic differential equations [27–29], and the control of dynamics [30–32]. While neural network offers successful alternative ideas to tackle difficult obstacles faced by conventional approaches, such as high dimensionality, adapting the neural network to a practical setting is still a highly nontrivial task.

While several efforts have taken a data-driven approach to test deep reinforcement learning to the control of quantum systems [33, 34], the application of neural network with a model-driven approach is still very new [35, 36]. Our work in this paper aims to expand the existing knowledge in this field by combining a neural network approach with a sampling-based learning method [37] to control a time-dependent quantum system with uncertainty. The goal is to find a control to prepare an initial state through the evolution of the quantum system to achieve a pre-selected targets state $|\psi_{target}\rangle$, while the quantum system contains uncertain parameters that satisfy known probability distributions.

A typical sampling-based method consists of two phases: "training" and "test". We incorporate both phases into the training process of the neural networks. We first set up neural network models to learn the controls of the system, with the uncertainty parameter included in the design of the loss function. In the training phase, we train the neural network for fixed training steps with one sample set of the uncertainty parameters generated from the given distribution. We then test the learned neural network for the control by evaluating its average performance on additional sample sets of the parameters, computed using standard finite difference method. In other words, the neural networks are trained with the loss function modified by a different set of sampled uncertainty parameter after every few fixed training steps, until the test fidelity stop improving. We also develop two ways to construct the neural networks: (1) a "Neural network only" ap-