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Adaptive Fully Implicit Simulator with Multilevel Schwarz Methods for Gas Reservoir Flows in Fractured Porous Media

Rui Li[1], Haijian Yang[1,*] and Chao Yang[2,*]

[1] *School of Mathematics, Hunan University, Changsha, Hunan, 410082, P.R. China.*
[2] *School of Mathematical Sciences, Peking University, Beijing, 100871, P.R. China.*

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Abstract. Large-scale reservoir modeling and simulation of gas reservoir flows in fractured porous media is currently an important topic of interest in petroleum engineering. In this paper, the dual-porosity dual-permeability (DPDP) model coupled with the Peng-Robinson equation of state (PR-EoS) is used for the mathematical model of the gas reservoir flow in fractured porous media. We develop and study a parallel and highly scalable reservoir simulator based on an adaptive fully implicit scheme and an inexact Newton type method to solve this dual-continuum mathematical model. In the approach, an explicit-first-step, single-diagonal-coefficient, diagonally implicit Runge–Kutta (ESDIRK) method with adaptive time stepping is proposed for the fully implicit discretization, which is second-order and L-stable. And then we focus on the family of Newton–Krylov methods for the solution of a large sparse nonlinear system of equations arising at each time step. To accelerate the convergence and improve the scalability of the solver, a class of multilevel monolithic additive Schwarz methods is employed for preconditioning. Numerical results on a set of ideal as well as realistic flow problems are used to demonstrate the efficiency and the robustness of the proposed methods. Experiments on a supercomputer with several thousand processors are also carried out to show that the proposed reservoir simulator is highly scalable.

AMS subject classifications: 65F08, 49M15, 65M55, 68W10, 76S05, 90B05
Key words: Reservoir simulation, fully implicit method, multilevel method, restricted Schwarz preconditioners, parallel computing.

1 Introduction

As important energy resources in the world, unconventional oil and natural gas in naturally fractured porous media have been receiving growing attentions in petroleum en-

*Corresponding author. *Email addresses:* lirui319@hnu.edu.cn (R. Li), haijianyang@hnu.edu.cn (H. Yang), chao_yang@pku.edu.cn (C. Yang)

gineering [11, 16]. The corresponding multiphysics flow problem not only inherits the difficulties of conventional reservoirs, such as the complex geological heterogeneity, the discontinuity of the medium and the computation of Darcy's law, but also exhibits significantly additional challenges when the fracture and the gas compressibility are involved in. In fractured reservoirs, mathematical models need to describe the transport of gas between the matrix blocks and the fractures with dual-mechanism such as dual-porosity dual-permeability [1, 11, 15]. Compared to the matrix pore, the fractures usually have higher porosities and permeabilities, but only occupy small volumes of the reservoir region. In such circumstances, the simulation should admit long periods of transport process and multi-scale complicated terrain, which often result in higher nonlinearities in the governing equations of fluid flow. As a result, to capture highly temporally and spatially varying numerical solutions, high resolution grids are often required to represent complex geological fluid physics, and reservoir simulators that can efficiently make use of massively parallel computers are necessary. Additional computational issue comes from the efficient simulation of compressible gas flow in porous media. Due to the compressibility of natural gas, the Peng–Robinson equation of state (PR-EoS) [10, 22, 23, 42] needs be coupled with the governing equations of fluid flow. The mechanisms between the fluid flow and the PR-EoS coexist and interplay simultaneously during the long span of the whole process, which brings a higher nonlinearity of the flow system and poses persistent challenges on the computational methodology. Hence, in this study we address the encountered challenges by designing a parallel and highly-efficient reservoir simulator, based on an adaptive fully implicit temporal scheme and a domain decomposition based solver.

Many temporal discretization schemes can be used to integrate the reservoir flow problems. Among them, the family of fully implicit methods is becoming one of the most promising schemes in the scientific community and the industrial world for extreme-scale simulations [10, 11, 16, 27, 29]. Compared to explicit or semi-implicit schemes, fully implicit methods typically enjoy two major advantages: (a) the spatial and temporal mesh sizes for reservoir simulation mostly depend on the accuracy requirement, rather than the stability constraint; (b) the discretized equations are solved simultaneously and implicitly, thus can better inherit the characteristics of the original mathematical model. In particular, the fully implicit approach can be combined with some suitable time-stepping strategies so as to substantially improve the accuracy of simulation, and this situation becomes more obvious when the high resolution simulation with multiple space and time scales is of interest, which greatly broadens the scope of the fully implicit applications. Recently, the fully implicit temporal scheme has been successfully utilized to flow through porous media models, such as the reactive transport problem [9, 24], multiphase fluid flows [28, 37, 39, 41, 43], and the compositional flow model [3, 31]. However, many attempts for reservoir simulation are based on the backward Euler method, which is L-stable but is only first-order accurate. In particular, neither of these temporal discretization schemes is efficient or accurate enough to be incorporated with an adaptive time stepping, and a further investigation of higher order temporal discretization schemes is