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A Colocalized Scheme for Three-Temperature Grey Diffusion Radiation Hydrodynamics

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Abstract. A positivity-preserving, conservative and entropic numerical scheme is presented for the three-temperature grey diffusion radiation hydrodynamics model. More precisely, the dissipation matrices of the colocalized semi-Lagrangian scheme are defined in order to enforce the entropy production on each species (electron or ion) proportionally to its mass as prescribed in [34]. A reformulation of the model is then considered to enable the derivation of a robust convex combination based scheme. This yields the positivity-preserving property at each sub-iteration of the algorithm while the total energy conservation is reached at convergence. Numerous pure hydrodynamics and radiation hydrodynamics test cases are carried out to assess the accuracy of the method. The question of the stability of the scheme is also addressed. It is observed that the present numerical method is particularly robust.

AMS subject classifications: 65M12, 35Q35, 82D10, 82A25

Key words: Colocalized Lagrangian scheme, radiation hydrodynamics, grey diffusion, discrete entropy production, plasma physics simulations.

Introduction

Background

Considering a non-equilibrium plasma, the relaxation process to reach the electronic and ionic temperature equalization occurs on time scales much shorter than the ones involved to reach quasi-neutral regimes or electron and ion Maxwellian equilibrium distribution functions [7]. This must be highlighted since in many practical applications, such as astrophysics or inertial confinement fusion, the characteristic times of interest can be of the same order of the temperature relaxation times. When this is the case, a two-temperature

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hydrodynamics model is required [7]. In the presence of strong radiation fields, the modeling of photon transport is also mandatory. Various physical descriptions are available depending on the level of accuracy needed [22]. Here, we restrict ourselves to the study of a three-temperature grey diffusion radiation hydrodynamics model, which is only valid in the optically thick limit [22].

The numerical resolution of radiation hydrodynamics models has been heavily investigated over the years. If one requires a transport-type modeling, among other methods to simulate thermal radiation propagation is the Implicit Monte-Carlo method. Early investigations may be found in [12] where the Implicit Monte-Carlo method is used for a two-temperature (radiation and matter temperatures) model. See also [9] in which the Implicit Monte-Carlo method is extended to the study of three-temperature models. We also mention here that angular moments (PN) and discrete ordinate (SN) methods are also widely used in this context [4, 22]. If the physical problem studied allows diffusion-type approximations, i.e if the photon mean free path is very small compared to the characteristic length of the problem (optically thick media) [13, 25], then standard Newton-Raphson or fix point strategy are also widely used to solve three-temperature models. Here it is assumed that the photon spectrum is Planckian and the radiation field is determined by many absorptions and re-emissions. In this case the radiation field rapidly becomes Planckian at a temperature not necessarily same as the material. We refer to [10] for numerical comparisons between several simulation codes for solving three-temperature models. It should also be mentioned that Jacobian-free Newton-Krylov methods enable the derivation of efficient algorithms for radiation diffusion equations [11,24]. Finally, notice that the time integration strategy of non-equilibrium radiation diffusion models has been investigated in [16] and references therein. We also mention here that purely diffusive model has a limited applicability and flux-limiting techniques are usually used [25]. The integration of limiting techniques to the numerical strategy presented in this document does not pose any issue and is not mentioned.

The numerical resolution of three-temperature models has become an active research field in recent years. In [29, 30] the numerical resolution of a three-temperature radiation hydrodynamics model on unstructured grids is presented. Concerning the numerical resolution of diffusion operators on (strongly) deformed meshed we refer to [1,3,26] and the references therein. We also mention that very recent studies have been published, specifically dealing with the positive preserving (or maximum principle preserving) properties for three-temperature radiation diffusion equations [27, 33].

In the present study, we do not consider pure Eulerian strategies. Indeed, even if large material deformation may be handled by Euler codes, their diffusive feature makes multi-material configurations difficult to handle. Of course, theses issues may be addressed with advanced material interface tracking techniques [28] inside mixed cells. However and more importantly, the discrete entropy production (per species) is almost never studied (or even mentioned) when working with pure Euler formalism. Gibb's relation cannot be easily combined with the evolution equations so that this key point is often neglected. Even if one could expect a global entropy production by the scheme, the