

Mesh Adaptation for Curing the Pathological Behaviors of an Upwind Scheme

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Abstract. In present paper, mesh adaptation is applied for curing the pathological behaviors of the enhanced time-accurate upwind scheme (Loh & Jorgenson, AIAA 2016). In the original ETAU (enhanced time-accurate upwind) scheme, a multi-dimensional dissipation model is required to cure the pathological behaviors. The multi-dimensional dissipation model will increase the global dissipation level reducing numerical resolution. In present work, the metric-based mesh adaptation strategy provides an alternative way to cure the pathological behaviors of the shock capturing. The Hessian matrix of flow variables is applied to construct the metric, which represents the curvature of the physical solution. The adapting operation can well refine the anisotropic meshes at the location with large gradients. The numerical results show that the adaptation of mesh provides a possible way to cure the pathological behaviors of upwind schemes.

AMS subject classifications: 65M08, 65M12, 65M55

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1 Introduction

For finite volume methods, upwind schemes are very popular due to their robustness and geometric flexibility. Following Godunov's idea [1], the Godunov type upwind schemes

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[2–4] applied a Riemann solver for flux calculations. However, they contain some cases of pathological behaviors, including the carbuncle phenomenon, the kinked Mach stem, the expansion shock and the numerical oscillation across a slow moving shock etc. [5–7].

The most renowned pathological behavior is the carbuncle phenomenon. It is firstly observed by Peery and Imlay [6]. The carbuncle phenomenon refers unphysical solutions at the stagnation region for computation of a supersonic flow over a blunt body. Many efforts have been made to study the origin of the carbuncle phenomenon. Through linear analysis, Sanders et al. [8] suggested the anomaly is the result of inadequate cross flow dissipation. Robinet et al. [9] pointed out that the carbuncle phenomenon is linked to the intrinsic instability of the Euler equations. By linear stability analysis, Gressier et al. [10] suggested the odd-even decoupling contributes the failures in some situations. By matrix stability analysis, Dumbser et al. [11] confirmed the problem of the odd-even decoupling. According to Xu et al. [12, 13], the dissipative mechanism of Godunov-type schemes is mainly from the averaging stage. In the direction parallel to the normal shock, the velocity is almost the same for two neighboring mesh cells, the numerical dissipation introduced in the averaging process diminishes. Without enough dissipation, the perturbation of pressure field will feed the density field, which is likely to induce shock instability. Kemm [14] also identified that low numerical viscosity on shear waves is the source of carbuncle. It is generally agreed that the insufficient dissipation is responsible for the pathological behaviors. Various methods have been proposed to cure the pathological behaviors. For instance, Quirk [7] used a more dissipative HLLC scheme near the shock. Lin [15] proposed a cure of Roe's scheme by reducing second-order accuracy to first-order accuracy inside the shock layer. Kim et al. [16] proposed a function to control the damping characteristic of Roe's scheme. Loh and Jorgenson [17] proposed a multi-dimensional dissipation model to systematically cure the pathological behaviors. However, these approaches will increase the dissipation level, thus reducing the numerical resolution, increasing the shock thickness or smearing some features of the flow field. Recently, Fleischmann et al. [18, 19] proposed a novel approach that cures the carbuncle phenomenon by reducing numerical dissipation instead of increasing it. They suggested that the inadequate scaling of the acoustic dissipation in the low Mach number limit causes the instability. By modifying the eigenvalue calculation in the Roe approximation, the approach can prevent the grid-aligned shock instability.

On the other hand, according to Quirk's observation [7], the alignment of the grid line to the shock is an essential condition for triggering the pathological behaviors. Meanwhile, Pandolfi and D'Ambrosio [20] proposed that the grid aspect ratio has significant influence to the carbuncle phenomenon. The elongated elements along the normal of the shock promote the instabilities, whereas elements stretched along the tangent to the shock have a damping effect. Recently, Rodionov [21] suggested that the grid alignment with the flow lines at the shock front is critical for the activation of the physical anomaly. He also pointed out that the numerical instability is sensitive to the size and shape of the grid cells.

These works inspired us to cure the pathological behaviors from the aspect of mesh