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On a Hybrid Approach for Recovering Multiple Obstacles

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Abstract. In this paper, a hybrid approach which combines linear sampling method and the Bayesian method is proposed to simultaneously reconstruct multiple obstacles. The number of obstacles and the approximate geometric information are first qualitatively obtained by the linear sampling method. Based on the reconstructions of the linear sampling method, the Bayesian method is employed to obtain more refined details of the obstacles. The well-posedness of the posterior distribution is proved by using the Hellinger metric. The Markov Chain Monte Carlo algorithm is proposed to explore the posterior density with the initial guesses provided by the linear sampling method. Numerical experiments are provided to testify the effectiveness and efficiency of the proposed method.

AMS subject classifications: 62F15, 78A46, 65N21, 35R30

Key words: Inverse scattering, multiple obstacles, linear sampling method, Bayesian method, hybridization.

1 Introduction

In this paper, we are mainly concerned with the inverse scattering problem of recovering multiple impenetrable obstacles due to plane wave incidence at a fixed frequency. This is a prototypical model for many industrial and engineering applications including radar/sonar, geophysical exploration and medical imaging. In what follows, we first present the mathematical setup for our study.

Suppose that D is a bounded domain in $\mathbb{R}^2$ such that $D=\bigcup_{i=1}^{I} D_i$, $\overline{D_i}\bigcap\overline{D_j}=\emptyset$ $(i\neq j)$. Each scatterer D_i is a simply connected domain with a C^2 smooth boundary ∂D_i. The

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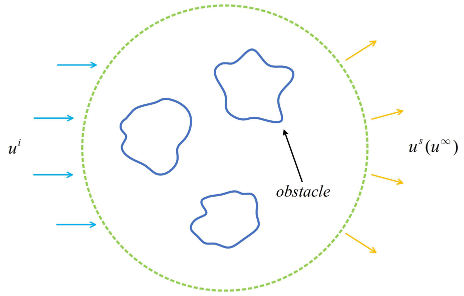

Figure 1: A schematic illustration of multiple obstacles scattering problem.

incident field is given by the time-harmonic plane wave of the form $u^i = e^{\mathrm{i}kx\cdot d}$, where $k\in\mathbb{R}_+$ is the wave number, $x\in\mathbb{R}^2$, $d\in\mathbb{S}:=\{x\in\mathbb{R}^2;\ |x|=1\}$ is the incident direction and $\mathrm{i}:=\sqrt{-1}$. See Fig. 1 for the illustration of scattering model. The direct scattering problem is to find total field $u=u^i+u^s$ generated by the incident field and the obstacle, which is governed by the following Helmholtz equation

$$\triangle u+k^2u=0 \quad \text{in } \mathbb{R}^2\backslash\overline{D}, \tag{1.1}$$

where u^s signifies the scattered field. On each obstacle, the total field u satisfies the following Dirichlet boundary condition

$$u=0 \quad \text{on } \partial D. \tag{1.2}$$

This boundary condition signifies that the pressure of total field vanishes on the boundary ∂D. Moreover, u^s satisfies the Sommerfeld radiation condition

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial u^s}{\partial r}-\mathrm{i}ku^s\right)=0, \quad r=|x|, \tag{1.3}$$

which characterizes the outgoing nature of the scattered field and holds uniformly in all directions. There exists a unique solution $u\in H^1_{loc}(\mathbb{R}^2\backslash\overline{D})$ for the Dirichlet exterior boundary value problem of (1.1)-(1.3) (cf. [9,26,32]). In addition, the scattered field u^s possesses the following asymptotic expansion

$$u^s(x)=\frac{e^{\mathrm{i}kr}}{\sqrt{r}}\left\{u^\infty(\hat{x},d)+\mathcal{O}\left(\frac{1}{r}\right)\right\}, \quad r=|x|\to\infty, \tag{1.4}$$

which holds uniformly for all observation directions $\hat{x}=x/|x|$. $u^\infty(\hat{x},d)$ is known as the far-field pattern of the scattered field $u^s(x)$. In the physical context, (1.1)-(1.3) describes the transverse electromagnetic scattering due to a infinitely long cylindrical-like conducting obstacle, where D signifies the cross section of the obstacle; see e.g. [4,16] for the related discussion.