Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2021-0136

Vol. **31**, No. 3, pp. 893-912
March 2022

Incorporating the Maximum Entropy on the Mean Framework with Kernel Error for Robust Non-Blind Image Deblurring

Hok Shing Wong[1], Hao Zhang[1], Lihua Li[2], Tieyong Zeng[1] and Yingying Fang[1,*]

[1] *Department of Mathematics, The Chinese University of Hong Kong, Hong Kong.*
[2] *College of Life Information Science and Instrument Engineering, Hangzhou Dianzi University, Hangzhou, China.*

Received 21 June 2021; Accepted (in revised version) 13 December 2021

Abstract. Non-blind deblurring is crucial in image restoration. While most previous works assume that the exact blurring kernel is known, this is often not the case in practice. The blurring kernel is most likely estimated by a blind deblurring method and is not error-free. In this work, we incorporate a kernel error term into an advanced non-blind deblurring method to recover the clear image with an inaccurately estimated kernel. Based on the celebrated principle of Maximum Entropy on the Mean (MEM), the regularization at the level of the probability distribution of images is carefully combined with the classical total variation regularizer at the level of image/kernel. Extensive experiments show clearly the effectiveness of the proposed method in the presence of kernel error. As a traditional method, the proposed method is even better than some of the state-of-the-art deep-learning-based methods. We also demonstrate the potential of combining the MEM framework with classical regularization approaches in image deblurring, which is extremely inspiring for other related works.

AMS subject classifications: 65J22, 65K10, 65T50

Key words: Image deblurring, total variation, KL divergence, error kernel.

1 Introduction

Image deblurring is a challenging ill-posed inverse problem. It is also a crucial step in many high-level imaging tasks such as object detection and pattern recognition. The

*Corresponding author. *Email addresses:* yyfang@math.cuhk.edu.hk (Y. Fang), hswong@math.cuhk.edu.hk (H. S. Wong), haozhang@math.cuhk.edu.hk (H. Zhang), lilh@hdu.edu.cn (L. Li), zeng@math.cuhk.edu.hk (T. Zeng)

blurring process is often modeled by the convolution of a blurring kernel with a ground truth image as follows

$$y=k*x+n, \tag{1.1}$$

where k denotes the blurring kernel, n denotes the image noise, and y and x represent the blurry and ground truth images respectively. Based on the accessibility of the kernel k, image deblurring can be categorized as blind deblurring and non-blind deblurring. In blind deblurring, the blurring kernel is unknown and needs to be estimated. While for non-blind deblurring, a kernel k is given as an input to recover the ground truth image x. In the current literature, many outstanding blind deblurring methods have been proposed to estimate both the clear image and the blurring kernel simultaneously [3, 5, 31]. Moreover, another type of method [6, 16, 18, 19, 25, 26] focuses on estimating the blurring kernel and subsequently apply the carefully chosen non-blind deblurring method to reconstruct the clear image. In this work, we first apply some blind deblurring methods to estimate the blurring kernels and mainly focus on recovering ground truth image x with the estimated kernels which could be inaccurate.

Due to the ill-posedness of the deblurring problem, the techniques of regularization have been widely imposed to ensure the robustness of deblurring methods with noise. The Maximum a Posterior (MAP) method is a standard tool to derive regularization terms by assuming certain image priors. The Tikhonov regularization [33] assumes the smoothness of the image. Moreover, ℓ_1-regularizer [2, 24] is a common regularization term in image deblurring. Indeed, the Total Variation (TV) method [13, 30] assumes the sparsity of the image gradients. Some other ℓ_1-norm-based regularizations extend this idea by assuming the sparse representation of image patches in some transform domains such as Discrete Cosine Transform (DCT) [23], wavelet [1, 9, 10] and framelet [4]. The hyper-Laplacian image prior [15] uses various ℓ_p-norms on image gradient as the regularization term. Furthermore, [8] is a nice example of non-local methods in which the image prior is imposed on image patches. Mathematically, the typical non-blind deblurring method with MAP-based regularization is formulated as

$$\min_x \left\{ \frac{1}{2}\|y-k*x\|_2^2+\lambda\phi(x) \right\}, \tag{1.2}$$

where ϕ is some regularization terms derived from the above image priors and λ is a positive parameter for balancing.

Variational models that are based on image priors and regularizations have remarkable performances. While a different paradigm of deblurring model is proposed recently. In [27, 28], Rioux *et al.* have proposed a new paradigm of deblurring model based on the principle of maximum entropy on the mean. The principle of maximum entropy states that the probability distribution which best represents the current state of knowledge is the one with the largest entropy. The principle of maximum entropy is used in [21, 22] to solve ill-posed problems in crystallography. It is then applied to deconvolution problems in astronomical imaging [20, 32, 34]. While the maximum entropy method considers