A DDG Method with a Residual-Based Artificial Viscosity for the Transonic/Supersonic Compressible Flow

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Abstract. In this work, a direct discontinuous Galerkin (DDG) method with artificial viscosity is developed to solve the compressible Navier-Stokes equations for simulating the transonic or supersonic flow, where the DDG approach is used to discretize viscous and heat fluxes. A strong residual-based artificial viscosity (AV) technique is proposed to be applied in the DDG framework to handle shock waves and layer structures appearing in transonic or supersonic flow, which promotes convergence and robustness. Moreover, the AV term is added to classical BR2 methods for comparison. A number of 2-D and 3-D benchmarks such as airfoils, wings, and a full aircraft are presented to assess the performance of the DDG framework with the strong residual-based AV term for solving the two dimensional and three dimensional Navier-Stokes equations. The proposed framework provides an alternative robust and efficient approach for numerically simulating the multi-dimensional compressible Navier-Stokes equations for transonic or supersonic flow.

1 Introduction

In recent decades, discontinuous Galerkin (DG) methods [1–5] have been widely developed to apply in computational fluid dynamics. DG methods are indeed a natural choice

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for solving the hyperbolic conservation laws, such as the compressible Euler equations. However, there are some challenges in developing the DG methods to solve the diffusion problems, such as the compressible Navier-Stokes equations, where the key issue is to construct viscous and heat fluxes. A number of methods have been proposed in the literature to address this issue, such as Interior penalty (IP) method [6, 7], symmetric IP (SIP) method [8], local discontinuous Galerkin (LDG) method [9], Bassi and Rebay (RR2) scheme [10], compact DG (CDG) scheme [11], hybridizable DG (HDG) scheme [12], recovery-based DG (RDG) method [13,14], reconstructed DG (rDG) method [15]. Besides those methods above, Gassner et al. [16] proposed a method based on the diffusive generalized Riemann problem (dGRP) to construct the numerical flux for diffusion problems. One can refer to the paper [17, 18] for more details of comparison among several of the methods above.

Recently, a direct discontinuous Galerkin (DDG) method has been widely investigated and applied in solving the Navier-Stokes equations. As the numerical flux defined by the DDG method is simple, compact, conservative, and consistent. Especially, the DDG method is more efficient in computation. The DDG method was first introduced by Liu and Yan [19,20] to solve diffusion problems, based on the direct weak formulation for solutions of parabolic equations. Later, Kannan and Wang [21] analyzed and optimized the DDG method based on Fourier analysis and successfully used it for two-dimensional (2-D) Navier-Stokes (N-S) equations based on spectral volume method. Their numerical experiments demonstrated that the results obtained by their DDG method are comparable to those by the LDG, BR2, and IP methods. In 2016, Cheng et al. [22] developed and extended the DDG method for solving the 2-D N-S equations on arbitrary grids. Moreover, Vidden and Yan [23] developed a symmetric DDG (SDDG) method for diffusion equations. In their work, a numerical flux for the test function derivative was introduced and more interface terms were added to DDG discretization. The SDDG method was extended and investigated to solve the compressible 2-D N-S equations by Yue et al. [24]. The numerical results show that the SDDG method can achieve the expected optimal order of accuracy. In 2018, Cheng et al. [25] developed an adjoint-based high-order h-adaptive direct discontinuous Galerkin method for 2-D steady state compressible N-S equations. In that work, the adjoint consistency was analyzed for SDDG, DDG, DDG with interface correction (DDGIC) and numerical results indicated its potential in the applications of adjoint-based adaptation for simulating compressible flows. Also, a parallel DDG method was developed to solved 3-D compressible Navier-Stokes equations on 3-D hybrid grids [26]. In 2018, Zhang et al. [27] developed the DDG method for the incompressible N-S equations on arbitrary grids with a simplified artificial compressibility flux. In the 54th AIAA Aerospace Sciences Meeting, Yang et al. [28] proposed a robust implicit and efficient DDG method to solve compressible Reynolds-averaged N-S (RANS) equations for simulating the compressible turbulent flows with Spalart-Allmaras (SA) turbulence model. Especially, an exact Jacobian was derived analytically from the Harten-Lax-van Leer-Contact (HLLC) convective flux accompanied with DDG viscous flux. In 2020, Shao et al. [29] combined the DDG method and DG/FV [30–33] hybrid