

Directional \mathcal{H}^2 Compression Algorithm: Optimisations and Application to a Discontinuous Galerkin BEM for the Helmholtz Equation

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Abstract. This study aimed to specialise a directional \mathcal{H}^2 (\mathcal{DH}^2) compression to matrices arising from the discontinuous Galerkin (DG) discretisation of the hypersingular equation in acoustics. The significant finding is an algorithm that takes a DG stiffness matrix and finds a near-optimal \mathcal{DH}^2 approximation for low and high-frequency problems. We introduced the necessary special optimisations to make this algorithm more efficient in the case of a DG stiffness matrix. Moreover, an automatic parameter tuning strategy makes it easy to use and versatile. Numerical comparisons with a classical Boundary Element Method (BEM) show that a DG scheme combined with a \mathcal{DH}^2 gives better computational efficiency than a classical BEM in the case of high-order finite elements and hp heterogeneous meshes. The results indicate that DG is suitable for an auto-adaptive context in integral equations.

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1 Introduction

Wave propagation and scattering phenomena appear in many fields of science and engineering. They are essential in geoscience, petroleum engineering, telecommunications, defence industry and acoustics. The simplest model of a wave scattered by an object is

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the famous Helmholtz equation, the governing equation of acoustics in a homogeneous medium which also indirectly arises in more complex wave models in electromagnetism or elastodynamics. One main difficulty induced by this class of equations is that the propagation domain is generally very large or unbounded. In the case of a homogeneous medium, the integral equation's formalism permits overcoming this difficulty. It consists in transforming the initial Helmholtz equation into an equivalent integral equation with its unknowns being potentials located on the surface of the scattering object. In our study, we choose to work in the scope of Boundary integral equations.

Real-life problems entail extensive propagation domains with complex geometry. This situation naturally implies many unknowns, which can be up to several million for an industrial problem. Moreover, determining the optimal approximation space regarding a prescribed accuracy is far from trivial and requires special tools. In this context, several approaches emerged in the BEM community to tackle these difficulties. Various authors [1, 7, 12, 18] developed *a posteriori* error estimates for the integral equation that yield a way to use an auto-adaptive loop refinement architecture. This approach constitutes an elegant methodology that generates the optimal approximation space regarding a prescribed accuracy. The existing studies were conducted using a classical Boundary Finite element method. The problem of the non-conforming approximation of integral operators – *i.e.* Discontinuous Galerkin (DG) scheme – began to receive the community's interest in the last years [10, 19, 25, 26]. This progress could contribute to improve the efficiency of these auto-adaptive loops notably. Indeed, the absence of conforming constraints allows the mesh to have great flexibility. It enables to work with hanging nodes, to locally use high order elements where the solution and the geometry are smooth. Using a DG scheme also eases the mesh generation process [33] (generation of a complex mesh per part and fusion). In a previous article [31], we proposed a theoretical and numerical study of a *hp* Interior Penalty Discontinuous Galerkin (IPDG) to discretise the hypersingular operator for high-frequency problems. In the present study, we aim to conceive a specialised matrix compression algorithm for the DG scheme introduced in [31], suitable within the context of an auto-adaptive loop. This problem is not trivial. Indeed the compression has to be efficient in the case of stiffness matrices coming from highly *hp* non-conforming and locally refined meshes and in low and high-frequency regimes. Among compression techniques reported in the extensive available literature, we focus on adapting the directional \mathcal{H}^2 from [6] in the case of a DG solver. Indeed this methodology possesses an interesting ratio between simplicity and efficiency while being naturally suitable for low and high-frequency problems. However, the particular shape of the DG matrix makes inefficient a straightforward transpose of the initial compression algorithm from [6]. Some original modifications will be necessary to implement in order to obtain an efficient and simple to use algorithm.

We have organised the rest of this paper in the following way: Section 2 briefly recalls the problem model and the Interior Penalty Discontinuous Galerkin (IPDG) scheme. In Section 3, we expose the specialised directional \mathcal{H}^2 -matrix for the DG scheme, the optimisations of the initial algorithm and the complexity estimates that quantify the impact