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An Adaptive Moving Mesh Method for the Five-Equation Model

Yaguang Gu[1,2], Dongmi Luo[3], Zhen Gao[1,*] and Yibing Chen[3]

[1] *School of Mathematical Sciences, Ocean University of China, Qingdao, Shandong, China.*
[2] *College of Oceanic and Atmospheric Sciences, Ocean University of China, Qingdao, Shandong, China.*
[3] *Institute of Applied Physics and Computational Mathematics, Beijing, China.*

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Abstract. The five-equation model of multi-component flows has been attracting much attention among researchers during the past twenty years for its potential in the study of the multi-component flows. In this paper, we employ a second order finite volume method with minmod limiter in spatial discretization, which preserves local extrema of certain physical quantities and is thus capable of simulating challenging test problems without introducing non-physical oscillations. Moreover, to improve the numerical resolution of the solutions, the adaptive moving mesh strategy proposed in [Huazhong Tang, Tao Tang, Adaptive mesh methods for one- and two-dimensional hyperbolic conservation laws, SINUM, 41: 487-515, 2003] is applied. Furthermore, the proposed method can be proved to be capable of preserving the velocity and pressure when they are initially constant, which is essential in material interface capturing. Finally, several classical numerical examples demonstrate the effectiveness and robustness of the proposed method.

AMS subject classifications: 35L65, 65M08, 76T10

Key words: Multi-component flows, five-equation model, finite volume method, minmod limiter, adaptive moving mesh method, stiffened gas EOS.

1 Introduction

Numerical study of the five equation model of two-component flows, proposed in [1], has been attracting much attention during the past twenty years, due to its wide range of applications in inertial confinement fusion, underwater explosion, shock bubble dynamics,

*Corresponding author. *Email addresses:* guyaguang@ouc.edu.cn (Y. Gu), dongmiluo@stu.xmu.edu.cn (D. Luo), zhengao@ouc.edu.cn (Z. Gao), chen_yibing@iapcm.ac.cn (Y. Chen)

and so on. Much work can be seen in the literature, such as finite volume method with high order weighted essentially non-oscillatory (WENO) reconstructions [5], discontinuous Galerkin (DG) approaches [4, 28, 29], high order finite difference alternative WENO (AWENO) method [11]. Other auxiliary techniques for the five-equation model such as bound- and positivity-preserving limiters can be found in [4, 12, 29, 44]. These studies cover the fluids with ideal, stiffened, and Mie-Grüneisen equations of state (EOS).

In capturing physical structures of the fluid, such as rarefaction waves, contact discontinuities and shocks, a core issue is that the proposed method should not produce spurious oscillations. TVD (total variation diminishing) reconstruction is an effective way of remedying this issue [23, 39]. One such possibility is to use linear reconstruction with minmod slope limiter, as it is easy to implement in one and higher dimensions. The minmod limiter compares the candidate slopes and selects the one with minimum magnitude if they have same sign, otherwise it returns zero slope. Modified by the minmod limiter, linear reconstructions provide approximations of the related physical quantities at cell interfaces, such that their local extrema are well preserved. As a result, spurious oscillations are not produced. However, a limitation of this approach is that it achieves only second order of accuracy, and solutions would be relatively dissipative, compared to those computed by high order methods. To remedy this limitation, a straightforward idea is to use adaptive mesh methods, such as adaptive mesh refinement method [17–19, 24, 26, 40], or adaptive moving mesh method [13–15, 20, 22, 25, 27, 38, 43, 46] which will be considered in this paper. There are mainly two categories of the adaptive moving mesh methods. The one is that a new distribution of the grid points is computed prior to solution evolution step at each time step, and the solution is then evolved directly to the next time level on the new mesh grids. Related studies can be seen in [20, 27]. The other one is to separate the adaptive moving mesh step from the solution evolution step, which is adopted in this paper. In this approach, the solutions are updated by repeating three steps: 1) evolve the solution to the next time step, 2) update the distribution of the grid points based on the solutions to be adapted, 3) update the solutions onto the newly computed mesh grids, where the latter two steps form the adaptive moving mesh strategy.

To solve compressible multi-component flows, there are mainly two types of approaches: sharp interface methods and diffusive interface methods. The sharp interface methods include Lagrangian or Lagrangian-Eulerian methods [16, 42], front tracking methods [9], level set methods [2, 30], ghost fluid methods [8, 48], and so on. In these methods, the material interface is resolved prior to the discretization strategy designed for fluids separated by the material interface. The diffusive interface methods allow a small amount of artificial transition zone at material interface, due to unavoidable numerical diffusion. If the physical law is well defined in this diffusive zone, the existing classical methods for single-component model, such as Euler equations, would be activated. Readers may refer to [34] for a comprehensive review of diffusive interface methods.

A notable feature of the model is that the velocity and pressure should remain unchanged during computation once they are initially constants, which is referred to as