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A RKDG Method for 2D Lagrangian Ideal Magnetohydrodynamics Equations with Exactly Divergence-Free Magnetic Field

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Abstract. In this paper, we present a Runge-Kutta Discontinuous Galerkin (RKDG) method for solving the two-dimensional ideal compressible magnetohydrodynamics (MHD) equations under the Lagrangian framework. The fluid part of the ideal MHD equations along with z-component of the magnetic induction equation are discretized using a DG method based on linear Taylor expansions. By using the magnetic flux-freezing principle which is the integral form of the magnetic induction equation of the ideal MHD, an exactly divergence-free numerical magnetic field can be obtained. The nodal velocities and the corresponding numerical fluxes are explicitly calculated by solving multidirectional approximate Riemann problems. Two kinds of limiter are proposed to inhibit the non-physical oscillation around the shock wave, and the second limiter can eliminate the phenomenon of mesh tangling in the simulations of the rotor problems. This Lagrangian RKDG method conserves mass, momentum, and total energy. Several numerical tests are presented to demonstrate the accuracy and robustness of the proposed scheme.

AMS subject classifications: 76M10, 76N15, 76W05

Key words: Lagrangian RKDG method, ideal compressible MHD equations, Taylor basis, exactly divergence-free magnetic field, limiter.

1 Introduction

In this paper, we continue our recent numerical researches in [42, 43] for the ideal Lagrangian magnetohydrodynamic (MHD) equations. The MHD flows can be found in a

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variety of important physical problems related to magnetic field, such as inertial confinement fusion (ICF) driven by Z-pinch dynamic hohlraum, implosion of solid liner driven by pulsed power systems, collisions of flyer plates accelerated by intense magnetic field with the target material in shock loading experiments, and so on. The initial configurations of the loads used in these physical problems consist of a variety of materials with complex shapes. In the physical process which follows, a high current up to dozens of megamperes is delivered to the loads on a time scale of 100-600 ns and produce an intense magnetic field exceeding 1000 tesla. At this time, under the action of Lorentz force, the interactions between materials or between the material and the magnetic field will occur, which will lead to large deformation, slip, fracture and other very complex changes of the materials.

Apart from resolving the interactions of various waves arising in the MHD flows, the numerical simulation of these MHD problems need to accomplish two important and difficult tasks. The first one is to distinguish clearly the interface between two immiscible materials. The material interface is time-dependent and therefore unknown a priori. It's one of the most important components of the solution, and its shape is very complex due to the large deformation of materials. The simulation of the material interface is still one of the most difficult problems in the field of computational fluid dynamics. The second task is to handle the divergence-free constraint condition of the magnetic field. Although this constraint condition is trivial in the one-dimensional case, it is not easy to provide magnetic field approximations with zero divergence in the case of multi-dimensional space, especially when the shape of the mesh is not so regular. Many numerical examples and analyses show that the non-zero divergence of the approximated magnetic field can cause numerical instability or non-physical properties of the numerical solution.

Up to now, a large number of magnetohydrodynamic numerical schemes with high accuracy and resolution have been established (see Balsara [1,3], Brio [10], Dai [14], Evans [18], Han [21], Li [25], Powell [31], Stone [33, 35, 36] and the references). These schemes can resolve well large gradients, shock waves, contact discontinuities and shear layers in magnetic fluids. In order to deal with the zero divergence constraint condition of the magnetic field, many numerical methods have been proposed too. These numerical methods can be roughly divided into four categories: projection method [9], constrained transport method [18], 8-wave formula method [31], and hyperbolic divergence cleaning method [16]. These techniques have a good effect in eliminating the divergence error of magnetic field and have been widely used. In recent years, in order to further improve the simulation accuracy of MHD problems, people have carried out research on high-order methods and proposed some effective numerical schemes, among which the works of Balsara, Fengyan Li and so on are more impressive. Based on WENO interpolation, Balsara et al. proposed a series of high-order finite volume schemes for two-dimensional ideal MHD equations with global zero-divergence magnetic field [1,2,5,6]. For Cartesian meshes, Fengyan Li et al. proposed a series of discontinuous Galerkin (DG) methods for solving two-dimensional ideal MHD equations, which can satisfy locally or globally divergence-free constraint condition of magnetic field [19, 22–24].