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Fourier Collocation and Reduced Basis Methods for Fast Modeling of Compressible Flows

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Abstract. A projection-based reduced order model (ROM) based on the Fourier collocation method is proposed for compressible flows. The incorporation of localized artificial viscosity model and filtering is pursued to enhance the robustness and accuracy of the ROM for shock-dominated flows. Furthermore, for Euler systems, ROMs built on the conservative and the skew-symmetric forms of the governing equation are compared. To ensure efficiency, the discrete empirical interpolation method (DEIM) is employed. An alternative reduction approach, exploring the sparsity of viscosity is also investigated for the viscous terms. A number of one- and two-dimensional benchmark cases are considered to test the performance of the proposed models. Results show that stable computations for shock-dominated cases can be achieved with ROMs built on both the conservative and the skew-symmetric forms without additional stabilization components other than the viscosity model and filtering. Under the same parameters, the skew-symmetric form shows better robustness and accuracy than its conservative counterpart, while the conservative form is superior in terms of efficiency.

AMS subject classifications: 76M10

Key words: Projection-based reduced order modeling, Fourier collocation, artificial viscosity, compressible flow.

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1 Introduction

In practice, there are simultaneous demands on accuracy and cost of numerical tools for multi-discipline design processes of modern aircrafts. In the context of computational fluid dynamics (CFD), a common strategy is to simplify physical models to ensure affordable cost, such as Reynolds averaged Navier-Stokes (RANS) models [1,2]. However, due to the inadequacy of RANS models for separation flows and the high demand for the simulation of complex configurations, higher fidelity strategies are required, e.g. large eddy simulations and direct numerical simulations. However, these are often overwhelming for many-query cases. Reduced order models (ROMs) [3,4] are considered as a promising data-driven approach to achieve a good balance between accuracy and cost. In general, ROMs rely on certain data set to construct the reduced basis, and to extract the intrinsic dimensionality of the problem, based on which low-dimensional dynamical models that are both accurate and efficient are developed. A common approach for ROMs is the projection-based ROM (PROM), relying on a dimensionality reduction technique, such as proper orthogonal decomposition (POD) [5, 6] to transform high-dimensional data into a reduced representation (or generalized coordinates), which is then evolved during the online stage. The full order model (FOM) is then projected onto this orthogonal basis, reducing the number of equations to be solved. For the online stage, the ROM of reduced dimension is solved efficiently to evolve the generalized coordinates.

In general, the stability of PROM can not be guaranteed even when its FOM counterpart is stable [7]. There have been a number of techniques proposed to enhance the stability of the ROM. One explanation for the lost stability is that POD works as a low-pass filter, which filters out small-scale structures. However, although these small-scale structures account for a small percentage in terms of energy, they are crucial for the dissipation mechanism. As a result, instability may appear if they are removed [8]. Several approaches have been proposed to model the missing scales in the ROM, similar to the LES subgrid models [9–11]. A second approach augments the original system with additional dissipation [12]. Lucia et al. [13] proposed to add artificial viscosity terms to the right hand side of the linearized Euler equations, employing constant viscosity. In [14], a similar approach is applied to the viscous Burgers equation, where an artificial term, similar to the Smagorinsky model [15], is added. In [16], the authors add spectral vanishing viscosity (SVV) directly to the low-dimensional dynamic system of viscous flows by invoking an assumption of similarity between the Fourier space and POD modal space. Krath et al. [17] proposed to directly add the artificial viscosity term to the reduced system with the viscosity determined using an energy-based stability analysis. Other stabilization strategies are available in the literature, including rotating the projection subspace [18], stabilizing the inner product [7, 19], least-squares Petrov-Galerkin projection (LSPG) [20–23], using basis functions that resolve a greater range of physical scales [24], etc.

Many PROMs proposed in the literature aim to solve smooth flows. For example, there have been a number of successful applications of PROMs to separated flows. Favier