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A Semi-Implicit Fractional Step Method Immersed Boundary Method for the Numerical Simulation of Natural Convection Non-Boussinesq Flows

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Abstract. The paper presents a novel pressure-corrected formulation of the immersed boundary method (IBM) for the simulation of fully compressible non-Boussinesq natural convection flows. The formulation incorporated into the pressure-based fractional step approach facilitates simulation of the flows in the presence of an immersed body characterized by a complex geometry. Here, we first present extensive grid independence and verification studies addressing incompressible pressure-driven flow in an extended channel and non-Boussinesq natural convection flow in a differentially heated cavity. Next, the steady-state non-Boussinesq natural convection flow developing in the presence of hot cylinders of various diameters placed within a cold square cavity is thoroughly investigated. The obtained results are presented and analyzed in terms of the spatial distribution of path lines and temperature fields and of heat flux values typical of the hot cylinder and the cold cavity surfaces. Flow characteristics of multiple steady-state solutions discovered for several configurations are presented and discussed in detail.

AMS subject classifications: 5Q30, 76M12, 76R10

Key words: Natural convection non-Boussinesq flows, pressure-corrected immersed boundary method, multiple steady state solutions.

1 Introduction

The ability to accurately simulate natural convection flows is critical for a wide range of engineering applications, including cooling electronic equipment, minimizing heat losses

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in buildings, investigating atmospheric flows, modeling heat transfer, and preventing accidents in the nuclear industry, to name but a few. The methods typically utilized for the simulation of natural convection flows can be classified into two major groups, one relying on weakly compressible approximations and the other addressing the fully compressible flow, as extensively reviewed in [1]. The first group of methods treats the flow as incompressible. In these methods, the buoyancy is introduced by employing either the Boussinesq approximation, which accounts for density variations in the gravity term and may also account for variations in the thermophysical properties of the flow, or Gay-Lussac-type approximations, which account for density variations in both the gravity and advection terms. The second group uses fully compressible Navier Stokes (NS) and energy equations.

The majority of weakly compressible approximations, which were developed for the simulation of low-Mach-number compressible flows, are based on an asymptotic model. The asymptotic model for the simulation of thermally driven natural convection flows was formulated for the first time in [2] and is known in the literature as the classical low-Mach-number model. The key idea was to split the pressure field into a large, time-dependent thermodynamic part and a stationary part that includes extremely small spatial deviations. Such a decomposition was found to be applicable for the simulation of low-Mach-number thermally driven flows, as it provides the same order of magnitude for all the terms in the momentum and energy equations and eliminates acoustic waves. Significant progress in this field may be attributed to the works [3–5] and to the study [6], which employed algorithms based on finite differences and spectral methods, respectively, for the simulation of non-Boussinesq natural convection confined flows. A weakly compressible approximation was used in further simulations of numerous non-Boussinesq natural convection flows [7–14] to address various problems in physics and computational science. The common drawback of all weakly compressible approximations is that the results obtained for flows with a dominant hydrodynamic part (e.g., high velocity flows) maybe inaccurate, which means that a fully compressible approach must be used.

Fully compressible approximations for low-Mach-number flows typically employ either density-based or pressure-based solvers. Although density-based formulations were traditionally utilized for simulating high-speed compressible flows (neglecting viscous effects), efforts have been made to extend the applicability of these formulations to configurations in which viscous effects play a significant role. These configurations include laminar natural convection compressible flows [7, 15, 16], low-Mach-number injection flows [11], and natural convection flows in a laminar-turbulent transition regime [17]. A key feature of density-based solvers is that continuity, momentum, and all other transport equations are first solved in a fully coupled manner, and the pressure field is then derived from an equation of state. As a result, the coupled operator is typically ill conditioned when applied to low-Mach-number flows, whose treatment requires sophisticated numerical techniques, such as preconditioning and dual-time stepping. When the simulations are performed on high-resolution grids or applied to 3D flows, density-based