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A Gas-Kinetic Scheme for Collisional Vlasov-Poisson Equations in Cylindrical Coordinates

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Abstract. Many configurations in plasma physics are axisymmetric, it will be more convenient to depict them in cylindrical coordinates compared with Cartesian coordinates. In this paper, a gas-kinetic scheme for collisional Vlasov-Poisson equations in cylindrical coordinates is proposed, our algorithm is based on Strang splitting. The equation is divided into two parts, one is the kinetic transport-collision part solved by multiscale gas-kinetic scheme, and the other is the acceleration part solved by a Runge-Kutta solver. The asymptotic preserving property of whole algorithm is proved and it's applied on the study of charge separation problem in plasma edge and 1D Z-pinch configuration. Numerical results show it can capture the process from non-equilibrium to equilibrium state by Coulomb collisions, and numerical accuracy is obtained.

AMS subject classifications: 65M08, 65M06, 35Q99

Key words: Vlasov-BGK-Poisson equations, cylindrical coordinates, gas-kinetic scheme, asymptotic preserving property, Coulomb collisions.

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1 Introduction

Plasma is a mixed medium with positive, negative and neutral particles. The motion of charged particles is coupled with the evolution of electromagnetic field. The flow regime of plasma is more complex than neutral gas. Many plasma parameters, such as Debye length, plasma frequency and ion inertial length, etc are important on the description of plasma. Among these parameters, two parameters are important in characterizing the flow regimes of plasma, namely the Knudsen number Kn and the normalized Larmor

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radius. The Knudsen number indicates the collision intensity and the normalized Larmor radius indicates the magnetization of plasma. On the kinetic scale, the dynamics of plasma is described by kinetic equation and the averaged electromagnetic field effect is modeled to the order of the reciprocal of normalized Larmor radius, and the effect of Coulomb collisions is modeled to the order of the reciprocal of Knudsen number. In the rarefied regime with large Knudsen number, the plasma is governed by Vlasov equation. In the densely collisional regime, the plasma can be described by magnetic hydrodynamic equations.

The multiscale property brings difficulties in numerical simulation of plasma. Much attention is concentrated in collisionless case, i.e Vlasov equation. The Particle-In-Cell (PIC) method is a classical solver for Vlasov equation. It is a Lagrangian method tracing the motion of particles in the electromagnetic field (see [1–7]). With a proper way to handle collision term, it can simulate the collisional Vlasov equation (see [8, 9]), but statistical noise is still a big problem for PIC method. The semi-Lagrangian method is a popular continuum method to solve Vlasov equation, and has been applied in various plasma problems (see [10–19]). In this method, distribution function is calculated by following the characteristic line backward in time, and then interpolating the value at the feet of characteristics using the grid point values at the previous time step. High order methods by interpolation can give a satisfied resolution. Except semi-Lagrangian method, Discontinuous Galerkin (DG) finite element method [20–22], Hamiltonian splitting method [23], adaptive tensor method [24], Interpolated Differential Operator (IDO) method [24], spectral method [25] are also successful solvers for Vlasov equation.

To get more physical meaning solution for this multiscale problem, hybrid methods [26, 27] that connect the Vlasov solver in the rarefied regime and magnetic hydrodynamic solver in densely regime are instinct. Recently, a series of asymptotic preserving (AP) schemes have been developed for kinetic equation, which can preserve the collisionless and Euler regimes [28] or Navier-Stokes regime [29, 30]. Even with asymptotic preserving property, the cell size is still limited by the mean free path scale for accuracy consideration to capture dissipation solution. Another AP scheme namely the unified gas kinetic scheme (UGKS) has been developed to simulate multiscale transport problems [31–38]. The UGKS models the flow physics on the scale of mesh size. When the cell size is on the hydrodynamic scale UGKS recovers the hydrodynamic behavior, and when the cell size is on the scale less than the mean free path UGKS recovers the particle free transport. In the transitional regime, it provides a physically consistent numerical flux.

Solvers mentioned above are based on Cartesian coordinates, but many magnetized plasma configurations are axisymmetric, such as field-reversed configurations, Z-pinches, Hall thrusters, spheromaks, particle beams and Tokamak. It's more convenient to depict them in cylindrical coordinates than in Cartesian coordinates. Some works have been done to solve collisionless Vlasov equation in cylindrical geometry and applied on various configurations, such as the study on Hall thrusters [39, 40], wakefield accelerators [41, 42], pulsed-power configurations [43], Z-pinches [44, 45], modeling plasma edge dynamics [46], high frequency electrostatic waves in a uniform magnetic field [47].