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Two Physics-Based Schwarz Preconditioners for Three-Temperature Radiation Diffusion Equations in High Dimensions

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Abstract. We concentrate on the parallel, fully coupled and fully implicit solution of the sequence of 3-by-3 block-structured linear systems arising from the symmetry-preserving finite volume element discretization of the unsteady three-temperature radiation diffusion equations in high dimensions. In this article, motivated by [M. J. Gander, S. Loisel, D. B. Szyld, SIAM J. Matrix Anal. Appl. 33 (2012) 653–680] and [S. Nardean, M. Ferronato, A. S. Abushaikha, J. Comput. Phys. 442 (2021) 110513], we aim to develop the additive and multiplicative Schwarz preconditioners subdividing the physical quantities rather than the underlying domain, and consider their sequential and parallel implementations using a simplified explicit decoupling factor approximation and algebraic multigrid subsolves to address such linear systems. Robustness, computational efficiencies and parallel scalabilities of the proposed approaches are numerically tested in a number of representative real-world capsule implosion benchmarks.

AMS subject classifications: 65F10, 65N55, 65Z05

Key words: Radiation diffusion equations, Schwarz methods, algebraic multigrid, parallel and distributed computing.

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1 Introduction

Equations of radiation hydrodynamics (RHD) model the complicated fluid motion and energy transport processes with coupled momentum and energy exchanges in high-energy density regime [7, 20, 25]. The RHD computations play an important role in numerous fields including inertial confinement fusion (ICF), reentry vehicles and astrophysics. The energy transport processes, which describe the streaming, scattering and absorption of radiation waves propagating through the diversified background media, are often approximated by the flux-limited three-temperature (3-T) radiation diffusion equations. It should be noticed that the effects from the tanglesome multi-physics couplings, a large spectrum of the interacting spatio-temporal scales, the inherent highly nonlinear feature and the presence of hydrodynamic instabilities dictating hundreds of millions of grid cells with large deformations cause significant difficulties to the discretizations, linearization and solution algorithms. Implicit time-stepping routines are required to remove the constraint of time-step size, the method of frozen coefficients [17] is exploited to assess all nonlinear terms at the preceding iteration level while numerous nonlinear iterations are performed to some prescribed tolerance for the solution at the next time step, and the finite volume scheme is utilized for locally conservative considerations, resulting in a suite of unsymmetric but positive definite and quite ill-conditioned linear systems which must be solved at the cost of generally more than 80% of the total ICF simulation time.

Efficient and scalable methods and software libraries on the current generation of pre-exascale parallel computers are a pivotal technology for high-resolution and high-fidelity simulations and analyses [3]. Two major categories of methods may be used: direct methods and iterative methods. However, even though the well-engineered direct solvers [1, 8, 18] are highly robust, iterative solvers may be preferred because they generally require dramatically less storage, allowing them to tackle quite large problems for which the memory requirements of direct solvers are prohibitive. Given the matrix size and sparsity degree of these systems, Krylov subspace methods [26] are normally the method of choice, however, their performance needs to be boosted through appropriate preconditioners with minimum user interventions, ideally, no need of other matrices to devise a preconditioner.

Over the previous two decades, a substantial literatures concerning the monolithic, block (namely, physics-based) and combined preconditioned Krylov subspace solvers paired with certain adaptive strategies have been developed to tackle the challenging task mentioned above, e.g., [2, 4, 5, 16, 21, 29, 32–34, 37–40] just to cite a few. It is worth noting that a Schur complement to accelerate the convergence of the generalized minimal residual (GMRES) solver without restarting was advanced by Brown and Woodward [5], the theoretical and practical block lower and upper triangular preconditioners were developed and analyzed by Shu et al. [29], a relaxed physical factorization preconditioner was excogitated by Yue et al. [39], the physical-variable based coarsening two-level (PCTL) preconditioner was originally proposed by Xu, Mo and An [33] and, more recently, fur-