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High-Order Unified Gas-kinetic Scheme

Gyuha Lim[1], Yajun Zhu[2] and Kun Xu[1,2,3,4,*]

[1] *Department of Mechanical and Aerospace Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China.*
[2] *Department of Mathematics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China.*
[3] *Guangdong-Hong Kong-Macao Joint Laboratory for Data-Driven Fluid Mechanics and Engineering Applications, Hong Kong University of Science and Technology, Hong Kong, China.*
[4] *Shenzhen Research Institute, Hong Kong University of Science and Technology, Shenzhen, China.*

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Abstract. In this paper, we present a high-order unified gas-kinetic scheme (UGKS) using the weighted essentially non-oscillatory with adaptive-order (WENO-AO) method for spatial reconstruction and the two-stage fourth-order scheme for time evolution. Since the UGKS updates both the macroscopic flow variables and microscopic distribution function, and provides an adaptive flux function by combining the equilibrium and non-equilibrium parts, it is possible to take separate treatment of the equilibrium and non-equilibrium calculation in the UGKS for the development of high-order scheme. Considering the fact that high-order techniques are commonly applied in the continuum flow simulation with complex structures, and that the rarefied flow structure is usually smooth in the physical space, we apply the high-order techniques in the equilibrium part of the UGKS for the capturing of macroscopic flow evolution, and retain the calculation of distribution function as a second-order method, so that a balance of computational cost and numerical accuracy could be well achieved. The HUGKS has been validated by several numerical test cases, including sine-wave accuracy test, Sod-shock tube, Couette, oscillating Couette, lid-driven cavity and oscillating cavity flow. It is shown that the current method preserves the multiscale property of the original UGKS and obtains accurate solutions in the near continuum regimes.

AMS subject classifications: 65M08, 65D05, 76P05, 82B40

Key words: High-order reconstruction, two-stage fourth-order scheme, WENO-AO, micro flow.

*Corresponding author. *Email addresses:* `glim@connect.ust.hk` (G. Lim), `mazhuyajun@ust.hk` (Y. Zhu), `makxu@ust.hk` (K. Xu)

1 Introduction

The gas-kinetic scheme (GKS) is a hydrodynamic flow solver based on the kinetic model equations [2, 26, 30, 32]. With the Chapman-Enskog (CE) expansion [5], the GKS can recover the Navier-Stokes (NS) solutions, and it combines the upwind and central difference schemes automatically according to the numerical resolution of flow physics. In the flux function, both the normal and tangential spatial gradients are considered, which results in the multi-dimensional property of the GKS. However, the assumption of CE expanded distribution function constrains the application of the GKS only in the near continuum flow regime. In order to extend the scheme for non-equilibrium flow, unified gas kinetic scheme (UGKS) has been developed [31]. The UGKS is a multiscale flow solver based on the direct modeling of flow physics on the numerical mesh size and time step scale by Courant-Friedrichs-Lewy (CFL) condition [6] with a discretized particle velocity space, and it utilizes integral solution of the BGK-type model, such as Shakhov equation, for gas evolution and flux evaluation at a cell interface. With an adaptive multiscale flux function, the UGKS has multiscale property to fully recover the flow physics from kinetic scale to hydrodynamic scale. In comparison with the direct simulation Monte Carlo (DSMC) method [3], the most prevailing particle method in the community of rarefied flow simulation, the UGKS has advantages in the near continuum regime in terms of high efficiency and accuracy without statistical noises. This property makes the UGKS to be suitable for low speed slip and transition regime problem such microflow and micro-electro-mechanical system (MEMS) applications [12].

In recent years, many high-order methods have been developed in the computational fluid dynamics (CFD) and more accurate solutions are obtained than those from the first- and second-order numerical schemes. For the finite volume scheme, the essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) have been developed [11, 21], and there are many modified versions of WENO, such as WENO-JS [15], WENO-Z [4] and WENO with adaptive-order (WENO-AO) [1]. High-order GKS (HGKS) have also been developed by incorporating these WENO methods for spatial reconstruction [14, 23]. Furthermore, the existence of time derivative term in the flux function enables the GKS to provide a high-order time evolution solution with fewer stages. For instance, the two-stage fourth-order temporal discretization method has been applied in the HGKS [19, 24]. For the same fourth order accuracy, it only requires two stages in one time step evolution, and achieves better computational efficiency than the Runge-Kutta (RK) method.

Meanwhile, the high-order method for rarefied flow regime have also been investigated in recent years. The original DSMC has first-order accuracy due to the decoupling treatment of convection and collision terms. The higher-order DSMC method was constructed by improving the temporal accuracy of the collision term [13]. However, statistical noise hinders particle method to get more accurate solution. The discrete unified gas kinetic scheme (DUGKS) [9, 10] gets high-order solution with third-order accuracy for low speed isothermal rarefied flow simulation by employing two-stage method [29].