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Partitioned and Monolithic Algorithms for the Numerical Solution of Cardiac Fluid-Structure Interaction

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Abstract. We review and compare different fluid-structure interaction (FSI) numerical methods in the context of heart modeling, aiming at assessing their computational efficiency for cardiac numerical simulations and selecting the most appropriate method for heart FSI. Blood dynamics within the human heart is characterized by active muscular action, during both contraction and relaxation phases of the heartbeat. The efficient solution of the FSI problem in this context is challenging, due to the added-mass effect (caused by the comparable densities of fluid and solid, typical of biomechanics) and to the complexity, nonlinearity and anisotropy of cardiac consitutive laws. In this work, we review existing numerical coupling schemes for FSI in the two classes of strongly-coupled partitioned and monolithic schemes. The schemes are compared on numerical tests that mimic the flow regime characterizing the heartbeat in a human ventricle, during both systole and diastole. Active mechanics is treated in both the active stress and active strain frameworks. Computational costs suggest the use of a monolithic method. We employ it to simulate a full heartbeat of a human ventricle, showing how it allows to efficiently obtain physiologically meaningful results.

AMS subject classifications: 65M60, 74F10, 76Z05

Key words: Heart modeling, active mechanics, fluid structure interaction, monolithic algorithms, partitioned algorithms.

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1 Introduction

The aim of this paper is to provide for the first time a systematic review and comparison of different fluid-structure interaction (FSI) numerical coupling schemes in the context of cardiac hemodynamics. In particular, we consider partitioned fully-coupled and monolithic algorithms and we analyze their effectiveness in both systolic and diastolic phases, and with both active stress and active strain modeling frameworks for muscular contraction. We investigate the performance of the schemes during the different phases of the heartbeat, in order to assess to which extent they depend on the specific physical features of such phases, in particular the presence of active forces.

The human heart acts as a pump, driven by the electrical activation of its cells, whose purpose is to force the blood into the circulatory system, allowing the delivery of oxygen and nutrients to the whole body [66]. The feedback mechanism between blood and cardiac muscle is relevant in determining the cardiac function and its response to pathological conditions [97, 98]. Numerical simulations offer a valid tool for the investigation of this mechanism [87].

A large number of computational studies model the fluid-solid feedback in the heart only in terms of a zero dimensional, lumped model for the blood flow [12, 21, 57, 67], mostly focusing on the electromechanical processes [12,43,53,92,98]. Alternatively, three-dimensional models for the blood flow are one-way coupled to mechanical models, receiving as input results of mechanical simulations to prescribe the boundary displacement of the fluid domain, but without feedback from the three-dimensional fluid model to the solid [65,106,117]. While FSI models for cardiac valves have been extensively studied [11, 30, 36, 56, 61, 73, 104], three-dimensional fluid dynamics models of blood in the cardiac cavities are seldom two-ways coupled with mechanical models for the cardiac muscle [28,75,81,99,102,111,113], due to the inherent complexity and computational cost of FSI simulations.

In the context of biomechanics, solving FSI problems poses significant challenges on the stability and efficiency of the numerical solution, mainly because of the comparable densities of fluid and solid (resulting in the added mass effect [27]) and the anisotropy and nonlinearity of the constitutive laws [51, 58]. Appropriate schemes are required to enforce the fluid-solid coupling in a computationally efficient way.

The FSI numerical coupling schemes that have been proposed in the literature (see, e.g., [20, 29, 56, 59, 70]) can be roughly classified into partitioned loosely coupled (or explicit) schemes [18, 24, 25, 40, 49, 50, 52], partitioned fully coupled (or fixed-point, or implicit) schemes [14, 15, 27, 69, 70, 76, 78] and monolithic (or Newton-based) schemes [45,55,64,81,93,114,115]. The schemes differ significantly in their modularity and in the implementation effort that they require, and in terms of their performance [68, 70].

The effectiveness of FSI schemes in the context of vascular hemodynamics has been widely studied, e.g., in [14,15,68,70]. However, the benchmarks and test cases under consideration were mostly related to the flow of blood within large vessels, rather than to the flow within cardiac chambers. In the heart, the flow is mostly driven by the interaction of