Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2022-0080

Vol. **32**, No. 5, pp. 1257-1286
November 2022

New Insights on Convergence Properties of Peridynamic Models for Transient Diffusion and Elastodynamics

Zhikun Zhou[1], Xuhao Peng[1], Pan Wu[1], Ziguang Chen[1,2,*] and Florin Bobaru[3]

[1] *Department of Engineering Mechanics, School of Aerospace Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China.*
[2] *Hubei Key Laboratory of Engineering Structural Analysis and Safety Assessment, 1037 Luoyu Road, Wuhan, 430074, China.*
[3] *Department of Mechanical and Materials Engineering, University of Nebraska-Lincoln, Lincoln, NE, 68588-0526, USA.*

Received 15 March 2022; Accepted (in revised version) 17 September 2022

Abstract. Recent analytical solutions for peridynamic (PD) models of transient diffusion and elastodynamics allow us to revisit convergence of 1D PD models to their classical counterparts. We find and explain the reasons for some interesting differences between the convergence behavior for transient diffusion and elastodynamics models. Except for very early times, PD models for transient diffusion converge monotonically to the classical one. In contrast, for elastodynamic problems this convergence is more complex, with some periodic/almost-periodic characteristics present. These special features are investigated for sine waves used as initial conditions. The analysis indicates that the convergence behavior of PD solutions to the classical one can be understood in terms of convergence properties for models using the Fourier series expansion terms of a particular initial condition. The results obtained show new connections between PD models and their corresponding classical versions.

AMS subject classifications: 45K05, 65M12, 65M22, 74A70, 74H10
Key words: Convergence, peridynamics, nonlocal models, analytical solutions, transient diffusion, elastodynamics.

1 Introduction

Peridynamics (PD) is an integral-type nonlocal theory [1,2], which provides an alternative scheme to the partial differential equations (PDEs)-based classical continuum me-

*Corresponding author. *Email addresses:* zchen@hust.edu.cn (Z. Chen), zkzhou@hust.edu.cn (Z. Zhou), pengxh@hust.edu.cn (X. Peng), ppwu@foxmail.com (P. Wu), fbobaru2@unl.edu (F. Bobaru)

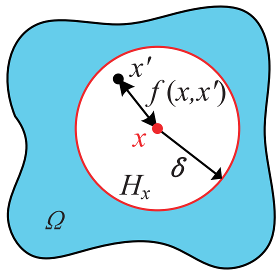

Figure 1: Schematic of peridynamic interactions for a node x with a horizon region (H_x) of radius δ in domain Ω.

chanics (CCM). In this theory, one essential feature is the nonlocality introduced by considering that any material point has "direct" interactions with its neighboring nodes over a certain region called "horizon", not only its nearest neighbors as in CCM. As shown in Fig. 1, material points, which interact with point x, within a distance δ of x form the "family" of x, H_x. Based on this nonlocality, the material mechanical response can be described using integrals over the horizon of a point. Spatial derivatives of deformation are therefore avoided, leading to excellent performance when modeling problems involving discontinuities, such as cracks, fracture, corrosion and dislocations, etc. [3–13]. Since its birth more than 20 years ago, PD has gained the attention of many researchers and its development has been rapid, in both theoretical developments and applications to a variety of fields [14–22]

A proper nonlocal theory is supposed to converge to the local counterpart in the limit of vanishing nonlocality. Efforts have been dedicated to investigate this convergence for PD models from a variety of viewpoints. Emmrich and Weckner [23] provided a rigorous study showing convergence of the bond-based Peridynamics (BBPD) to the classical Navier equation when the horizon goes to zero. Results concerning well-posedness and uniqueness were also presented there. In [14, 24], Silling and Lehoucq constructed the mathematical and conceptual relations between the nonlocal PD formulation and CCM. In addition, a number of mathematical [25–30] and numerical [31–35] studies were carried out to analyze the convergence of nonlocal PD models to the corresponding classical local ones.

In general, obtaining analytical solutions for nonlocal models (described by integro-differential equations) appears to be more challenging than finding them for the corresponding local models, based on partial differential equations. Recently, we have derived analytical solutions to the PD models for transient diffusion and dynamic elasticity in 1D and simple 2D domains with simple boundary conditions, by using the method of separation of variables, and obtaining solutions in the form of infinite series [36, 37]. With these PD analytical solutions, the convergence behavior of PD models to the local models can be investigated further. In this paper, we focus on investigating convergence properties of nonlocal solutions to the corresponding classical ones in 1D PD models of transient diffusion and elastodynamics.