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αSetup-PCTL: An Adaptive Setup-Based Two-Level Preconditioner for Sequence of Linear Systems of Three-Temperature Energy Equations

Silu Huang[1], Xiaoqiang Yue[2] and Xiaowen Xu[1,*]

[1] *Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China.*
[2] *National Center for Applied Mathematics in Hunan, Key Laboratory of Intelligent Computing and Information Processing of Ministry of Education, Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Xiangtan University, Xiangtan 411105, China.*

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Abstract. The iterative solution of the sequence of linear systems arising from three-temperature (3-T) energy equations is an essential component in the numerical simulation of radiative hydrodynamic (RHD) problem. However, due to the complicated application features of the RHD problems, solving 3-T linear systems with classical preconditioned iterative techniques is challenging. To address this difficulty, a physical-variable based coarsening two-level (PCTL) preconditioner has been proposed by dividing the fully coupled system into four individual easier-to-solve subsystems. Despite its nearly optimal complexity and robustness, the PCTL algorithm suffers from poor efficiency because of the overhead associated with the construction of setup phase and the solution of subsystems. Furthermore, the PCTL algorithm employs a fixed strategy for solving the sequence of 3-T linear systems, which completely ignores the dynamically and slowly changing features of these linear systems. To address these problems and to efficiently solve the sequence of 3-T linear systems, we propose an adaptive two-level preconditioner based on the PCTL algorithm, referred to as αSetup-PCTL. The adaptive strategies of the αSetup-PCTL algorithm are inspired by those of αSetup-AMG algorithm, which is an adaptive-setup-based AMG solver for sequence of sparse linear systems. The proposed αSetup-PCTL algorithm could adaptively employ the appropriate strategies for each linear system, and thus increase the overall efficiency. Numerical results demonstrate that, for 36 linear systems, the αSetup-PCTL algorithm achieves an average speedup of 2.2, and a maximum speedup of 4.2 when compared to the PCTL algorithm.

AMS subject classifications: 65F08, 65F10, 65N55, 65Z05

*Corresponding author. *Email addresses:* huangsilu1992@163.com (S. Huang), yuexq@xtu.edu.cn (X. Yue), xwxu@iapcm.ac.cn (X. Xu)

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1 Introduction

The numerical simulation of radiation hydrodynamics, which is described by several coupled equations, referred to as radiation hydrodynamics (RHD) equations, has long been an issue of great interest in a wide range of fields, such as the inertial confinement fusion (ICF), astrophysical phenomena and national defense security [1, 2]. The RHD equations describe the motion of fluids and radiative transition processes, and the latter is often expressed using three-temperature (3-T) energy equations under the assumption of diffusion approximation. Typically, the 3-T equations are discretized using the finite volume approach in space [3] and linearized using the Picard or Newton method [4], yielding a sequence of sparse linear systems whose count is surely into tens of thousands, hundreds of thousands, or even millions. In comparison to other components of RHD simulation, solving the 3-T linear systems is the most time-consuming part, accounting for 80% of total time [5]. To efficiently solve such sparse linear systems, preconditioned Krylov subspace iterative method is the most generally used approach, and the preconditioner has a considerable impact on the solver's efficiency and robustness.

Numerous academics have studied the preconditioners for 3-T linear systems [6–13], such as incomplete LU factorization (ILU), algebraic multigrid method (AMG), domain decomposition method (DDM) and their combinations. In [8], a physical-variable based coarsening algebraic two-level (PCTL) algorithm is proposed. The PCTL algorithm constructs a special coarse operator based on the underlying block structure of 3-T linear systems. Its main idea is to divide the fully coupled system into four individual scalar subsystems with the same nonzero pattern, which are substantially easier to solve. Due to the fact that the challenges emerging from the complicated couplings among physical quantities are addressed by utilizing the PCTL algorithm as a preconditioner, the convergence rate is significantly improved. However, the PCTL algorithm suffers from poor efficiency due to the high overhead associated with both the setup phase and the solve phase (including the solution of subsystems). Besides, when dealing with the sequence of 3-T linear systems whose properties are changed dynamically along with the dynamic evolution of application features, the PCTL algorithm completely neglects the different properties of the linear systems in the sequence and just utilizes fixed strategies for all systems, which also leads to a poor overall efficiency. Based on these points, we intend to propose an adaptive two-level setup-based preconditioner based on the PCTL algorithm for the sequence of 3-T linear systems, referred to as αSetup-PCTL. The αSetup-PCTL algorithm aims to utilize different strategies to solve the sequence of linear systems with dynamically changing properties. According to the earlier research and the associated numerical results [9], it is well established that the coupling strength and the