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The Effect of the Width of the Incident Pulse to the Dielectric Transition Layer in the Scattering of an Electromagnetic Pulse – a Qubit Lattice Algorithm Simulation

George Vahala[1,*], Linda Vahala[2], Abhay K. Ram[3] and Min Soe[4]

[1] *Department of Physics, William & Mary, Williamsburg, VA23185, USA.*
[2] *Department of Electrical & Computer Engineering, Old Dominion University, Norfolk, VA 12319, USA.*
[3] *Plasma Science and Fusion Center, MIT, Cambridge, MA 02139, USA.*
[4] *Department of Mathematics and Physical Sciences, Rogers State University, Claremore, OK 74017, USA.*

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Abstract. The effect of the thickness of the dielectric boundary layer that connects a material of refractive index n_1 to another of index n_2 is considered for the propagation of an electromagnetic pulse. A qubit lattice algorithm (QLA), which consists of a specially chosen non-commuting sequence of collision and streaming operators acting on a basis set of qubits, is theoretically determined that recovers the Maxwell equations to second-order in a small parameter ϵ. For very thin but continuous boundary layer the scattering properties of the pulse mimics that found from the Fresnel discontinuous jump conditions for a plane wave - except that the transmission to incident amplitudes are augmented by a factor of $\sqrt{n_2/n_1}$. As the boundary layer becomes thicker one finds deviations away from the discontinuous Fresnel conditions and eventually one approaches the expected WKB limit. However there is found a small but unusual dip in part of the transmitted pulse that persists in time. Computationally, the QLA simulations still recover the solutions to Maxwell equations even when this parameter $\epsilon \to 1$. On examining the pulse propagation in medium n_1, ϵ corresponds to the dimensionless speed of the pulse (in lattice units).

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*Corresponding author. *Email addresses:* gvahala@gmail.com (G. Vahala), lvahala@odu.edu (L. Vahala), abhay@psfc.mit.edu (A. K. Ram), msoe.rsu@gmail.com (M. Soe)

1 Introduction

For some time now [8–10, 15, 18, 19, 22–29, 32, 33], we have been developing qubit lattice algorithms (QLAs) as a computational scheme to efficiently solve certain nonlinear physics problems. QLA is a mesoscopic representation of a non-commuting set of interleaved collision and streaming operators on a basis of qubits which in the continuum limit perturbatively recovers the desired partial differential equations. To validate QLA, we [23–25] considered the exactly soluble one dimensional (1D) nonlinear Schrödinger equation (NLS)

$$i\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+|\psi|^2\psi=0.$$

In developing our QLA for 1D NLS we introduced 2 qubits, q_0 and q_1, per lattice site to represent the wave function ψ. We then determined a sequence of interleaved non-commuting unitary collision and streaming operators acting on this 2-qubit basis which in the continuum limit recovered the 1D NLS to second-order in a perturbation parameter ϵ. The unitary collision operator locally entangles the 2 qubits at that spatial site, while the unitary streaming operator moves this quantum entanglement throughout the lattice. In QLA simulations, the role of ϵ was the amplitude of the wave function $\psi=q_0+q_1$. Because of the symplectic structure of the algorithm, long-time integration of QLA successfully and with great precision [23–25] reproduced multiple soliton-soliton collision induced phase shifts. Because of the unitary structure of QLA there is some hope that the algorithm can be successfully encoded onto an error-correcting quantum computer, particularly when the quantum information science community solves the problem of how to encode nonlinearities (which in our 1D NLS QLA is the $|\psi|^2$-term).

Using the tensor products one can readily determine a QLA for the (non-integrable) 3D NLS and perform quantum turbulence simulations [26–28, 32, 33] for the time evolution of the ground state wave function for scalar Bose-Einstein Condensate (BECs). Like its distant cousin, the lattice Boltzmann algorithm, QLA is ideally parallelized on classical supercomputers and so we could perform long time integration to examine the triple energy cascade on a spatial grid of 5760^3, with 2 qubits/lattice site. Moreover, since QLA places low memory demands, this spatial grid was readily handled by using 11276 cores on a 2008 Cray supercomputer. It is interesting to note that the standard computational fluid dynamic (CFD) codes to simulate the 3D Hamiltonian BEC quantum turbulence required the introduction of a dissipative term (presumably to suppress numerical instabilities). To recover energy conservation at each time step, the CFD codes then required specific energy input terms at these time steps to counter the dissipative term. In contrast, the 3D BEC QLA algorithm preserved the Hamiltonian structure of the original equations and remained numerically stable.

We [15, 18, 19, 22, 29] then generalized the QLA to consider non-Abelian quantum vortices in spin-2 BECs. These spinor BECs consist of 5 coupled 3D Gross-Pitaevskii (i.e, NLS-like) equations and required just 10 qubits/lattice site. The parallelization of our QLA on Argonne's *MIRA* supercomputer saw no saturation with cores, even up to the