Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2022-0263

Vol. **33**, No. 3, pp. 692-732
March 2023

Fifth-Order A-WENO Path-Conservative Central-Upwind Scheme for Behavioral Non-Equilibrium Traffic Models

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Received 20 October 2022; Accepted 18 December 2022

Abstract. Non-equilibrium hyperbolic traffic models can be derived as continuum approximations of car-following models and in many cases the resulting continuum models are non-conservative. This leads to numerical difficulties, which seem to have discouraged further development of complex behavioral continuum models, which is a significant research need.

In this paper, we develop a robust numerical scheme that solves hyperbolic traffic flow models based on their non-conservative form. We develop a fifth-order alternative weighted essentially non-oscillatory (A-WENO) finite-difference scheme based on the path-conservative central-upwind (PCCU) method for several non-equilibrium traffic flow models. In order to treat the non-conservative product terms, we use a path-conservative technique. To this end, we first apply the recently proposed second-order finite-volume PCCU scheme to the traffic flow models, and then extend this scheme to the fifth-order of accuracy via the finite-difference A-WENO framework. The designed schemes are applied to three different traffic flow models and tested on a number of challenging numerical examples. Both schemes produce quite accurate results though the resolution achieved by the fifth-order A-WENO scheme is higher. The proposed scheme in this paper sets the stage for developing more robust and complex continuum traffic flow models with respect to human psychological factors.

AMS subject classifications: 76M20, 76M12, 65M06, 65M08, 76A30, 35L65, 35L67

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Key words: Finite-difference A-WENO schemes, finite-volume central-upwind schemes, path-conservative central-upwind schemes, non-oscillatory schemes, continuum traffic flow model, driver behavior.

1 Introduction

This paper is focused on the development of robust and highly accurate numerical methods for non-equilibrium continuum traffic flow models, as non-conservative systems of hyperbolic PDEs.

Continuum models treat traffic flow as a compressible fluid, and study its behavior using aggregated state variables (for instance, flow and density) and are useful for real-world traffic regarding operation and control [48]. Numerous continuum models have been developed over time to accommodate various empirical and behavioral aspects of traffic flow (see [47] for a critical review), which can be categorized into two broad families of equilibrium and non-equilibrium models.

Equilibrium models rely primarily on the differential forms of the mass conservation principle and some explicit functional forms between the state-variables (that is, speed and density). The most prominent example is the seminal Lighthill-Whitham-Richards (LWR) model [40, 54], which for a section of homogeneous road without intersections, can be presented as

$$\rho_t+(\rho V)_x=0,\quad V=V_e(\rho), \tag{1.1}$$

where x is the spatial variable, t is the time, $\rho(x,t)$ is the density, and $V_e(\rho)$ describes traffic speed as a generic function of local traffic density. The standard LWR model treats the multi-lane traffic as a single-pipe, assuming all vehicles and drivers have the same properties. Over time, numerous extensions of the LWR model have been proposed to incorporate various aspects such as multi-lane driving and lane-changing manoeuvres (see, e.g., [15,16,25,29]), different vehicle types (see, e.g., [4,49,53,61,62,64]), and drivers' non-local anticipation of traffic condition ahead (see, e.g., [5,10,11,57]).

Regardless of their underlying rationales, all equilibrium models are derived from the flow conservation principles, and thus, can always be presented in the form of the system of balance laws:

$$\boldsymbol{U}_t+\boldsymbol{F}(\boldsymbol{U})_x=\boldsymbol{S}(\boldsymbol{U}), \tag{1.2}$$

where $\boldsymbol{U}$ are the state variables, $\boldsymbol{F}$ are the nonlinear fluxes, and $\boldsymbol{S}(\boldsymbol{U})$ are the source terms. Therefore, equilibrium models are often solved using numerical methods for hyperbolic conservation and balance laws; see, e.g., [20,35,44,69].

Non-equilibrium models, on the other hand, use the same flow continuity equation as in (1.1), but with the speed adapted as a dynamic process. The majority of non-equilibrium models can be presented in the following generic form:

$$\begin{cases}\rho_t+(\rho V)_x=0,\\ V_t+VV_x=f(\rho,V,\rho_x,V_x,\cdots).\end{cases}$$