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# Implicit Quadrature-Free Direct Reconstruction Method for Efficient Scale-Resolving Simulations

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**Abstract.** The present study develops implicit physical domain-based discontinuous Galerkin (DG) methods for efficient scale-resolving simulations on mixed-curved meshes. Implicit methods are essential to handle stiff systems in many scale-resolving simulations of interests in computational science and engineering. The physical domain-based DG method can achieve high-order accuracy using the optimal bases set and preserve the required accuracy on non-affine meshes. When using the quadrature-based DG method, these advantages are overshadowed by severe computational costs on mixed-curved meshes, making implicit scale-resolving simulations unaffordable. To address this issue, the quadrature-free direct reconstruction method (DRM) is extended to the implicit DG method. In this approach, the generalized reconstruction approximates non-linear flux functions directly in the physical domain, making the computing-intensive numerical integrations precomputable at a preprocessing step. The DRM operator is applied to the residual computation in the matrix-free method. The DRM operator can be further extended to the system matrix computation for the matrix-explicit Krylov subspace method and preconditioning. Finally, the A-stable Rosenbrock-type Runge–Kutta methods are adopted to achieve high-order accuracy in time. Extensive verification and validation from the manufactured solution to implicit large eddy simulations are conducted. The computed results confirm that the proposed method significantly improves computational efficiency compared to the quadrature-based method while accurately resolving detailed unsteady flow features that are hardly captured by scale-modeled simulations.

**AMS subject classifications**: 76M10, 76N06, 76F65

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**Key words**: Direct reconstruction method, implicit Runge–Kutta method, quadrature-free method, discontinuous Galerkin method, implicit large eddy simulation, high performance computing.

## 1 Introduction

For the last few decades, the development of high-order computational fluid dynamics (CFD) methods has rapidly progressed, particularly based on finite element discretization techniques, owing to their attractive features [1]. Finite element-based high-order methods can achieve arbitrary high level of accuracy via local solution approximations on irregularly unstructured compact stencils [2]. This compactness greatly simplifies the message-passing process and enables high scalability on large-scale parallel computations [3, 4]. Compact stencils are also advantageous to complex mesh systems, leading to greater flexibility in dealing with complex configurations. The finite element-based high-order methods become more and more cost-competitive for a given level of accuracy as the approximation order increases, making them better suited for high-fidelity scale-resolving flow simulations [5]. The high-rank tensor operations widely used in finite element-based high-order methods are computationally intensive and can take full advantage of the high performance computing machines [6]. From this perspective, the finite element-based high-order methods will be the focus of this paper.

In high-order methods, numerical solutions are approximated by high-degree polynomials and represented by polynomial bases in each element. According to the basic formulation, a few classes of high-order methods are available [7–10]. In the present study, we focus on the physical domain-based modal discontinuous Galerkin (DG) method [11,12]. This method can provide the optimal order of accuracy using the optimal number of polynomial bases [13]. Sharing the same number of degrees of freedom in all elements allows simple and fast memory access when implemented on a computer. Compared to reference domain-based methods, it has no accuracy degradation on the meshes with non-affine elements [13, 14]; thus, the designed order of accuracy can be achieved on mixed-curved meshes. Using modal bases also enables further extensions to polyhedral meshes and makes the mass matrix identity [11, 15].

Despite their excellent performance and simplicity, high-order explicit formulations show a clear limitation in many scale-resolving simulations of practical applications because the allowable time-step size is too restrictive. In general, the CFL condition enforces $\Delta t \sim \mathcal{O}(\Delta x^2)$ in parabolic systems, such as the Navier–Stokes equations, which become very stiff when dealing with thin boundary layer meshes for high-Reynolds-number flows. The more stringent step size, $\Delta t \sim \mathcal{O}(\Delta x^2/k^2)$ where $k$ is a polynomial degree, in high-order methods aggravates the situation. An even more stringent time step has to be enforced to relax the numerical disturbances of the initial flow field. Typical examples are Reynolds-averaged Navier–Stokes (RANS) source terms and steep flow