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A Genuinely Two-Dimensional HLL-Type Approximate Riemann Solver for Hypo-Elastic Plastic Flow

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Abstract. In this work, a genuinely two-dimensional HLL-type approximate Riemann solver is proposed for hypo-elastic plastic flow. To consider the effects of wave interaction from both the x- and y-directions, a corresponding 2D elastic-plastic approximate solver is constructed with elastic-plastic transition embedded. The resultant numerical flux combines one-dimensional numerical flux in the central region of the cell edge and two-dimensional flux in the cell vertex region. The stress is updated separately by using the velocity obtained with the above approximate Riemann solver. Several numerical tests, including genuinely two-dimensional examples, are presented to test the performances of the proposed method. The numerical results demonstrate the credibility of the present 2D approximate Riemann solver.

AMS subject classifications: 35L45, 35Q35, 74C05, 74M20

Key words: Elastic plastic flow, elastic-plastic transition, multi-dimensional effect, two-dimensional approximate Riemann solver.

1 Introduction

Numerical simulations of weapon physics commonly involve elastic-plastic flow, including Armor Piercing (AP) projectiles [1–14]. The hypo-elastic plastic model is a suitable choice for accurately reproducing experimental data, particularly for metal materials, and can naturally introduce plastic deformation while easily handling complex multi-dimensional boundary problems.

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In the past decade, researchers have developed several approximate Riemann solvers for one-dimensional hypo-elastic plastic models for elastic-plastic flow using the popular Mie-Gru*ü*neisen equation of state [18–22]. Tang et al. [15] put forward an exact Riemann solver with the Murnagham equation of state. Recently, Gao et al. [24, 25, 56, 57] constructed the exact solver and approximate Riemann solvers by replacing the equation of state with the classic Hooke law in the elastic region, which was extensively studied in [23].

The one-dimensional solvers mentioned above have been extended to two-dimensional elastic-plastic flow using the dimension-splitting method. Cheng et al. [26, 27] proposed a four-rarefaction wave (FRRSE) and an HLLC approximate Riemann solver for two-dimensional elastic-plastic flows in a second-order cell-centered Lagrangian scheme. Li et al. [28] developed an HLLC-type approximate Riemann solver (HLLCN) that included more complex wave structures, such as elastic, plastic, longitudinal, and shear waves, simultaneously in the presence of elastic-plastic transition. David et al. [30] proposed a cell-centered Lagrangian method by hybridizing Lax-Wendroff and HLL methods for elastic-plastic flows. Maire et al. [16] developed a nodal two-dimensional approximate Riemann solver for elastic-plastic flows with simpler wave structures. Sambasivan et al. [17] extended that work to cylindrical axisymmetric geometries. Although the one-dimensional solver has been successfully extended to high dimensions, some researchers believe that the increase in dimensionality can introduce dimensional effects. In the case of the two-dimensional elastic-plastic flow studied in this paper, two-dimensional effects arise due to the presence of shear stress and total/deviator stress that are absent in one-dimensional systems. The constitutive model for stress in elastic-plastic flow is related to velocity gradients in different directions. Therefore, it is necessary to construct a genuinely two-dimensional Riemann solver that considers the interaction of nonlinear waves in different directions to showcase the two-dimensional effects in numerical results.

The two-dimensional Riemann problem (even for the most straightforward isotropic ideal gas) is so complicated that constructing a two-dimensional exact Riemann solver has yet to be fulfilled and seems impractical at present [31–36]. Based on their work, Lax and Liu [38] proposed positive schemes to solve Riemann problems for two-dimensional gas dynamics. Glimm et al. [37] presented numerical evidence and generalized characteristic analysis to establish the existence of a shock wave through the interaction of four rarefaction waves. Wendroff et al. [30, 40] extended the theory of one-dimensional HLL approximate Riemann solver to two dimensions and proposed a nine constant states Riemann solver, but its formulation lacks explicit expressions. Brio et al. [41] presented a multistate Riemann solver, which was only applicable to the Euler equations of gas. Vides et al. [42] introduced a simple two-dimensional nine-state HLL approximate Riemann solver, which approximately resolved the wave interacting region with only one constant state. Capdeville [39] devised a four-state bi-dimensional HLL solver (HLL-2D) on the triangular cell. Later, based on the previous work [39, 40, 43], Balsara [44] introduced a contact discontinuity at the wave interacting region and presented a two-dimensional