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Learning Invariant Representation of Multiscale Hyperelastic Constitutive Law from Sparse Experimental Data

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Abstract. Constitutive modeling of heterogeneous hyperelastic materials is still a challenge due to their complex and variable microstructures. We propose a multiscale data-driven approach with a hierarchical learning strategy for the discovery of a generic physics-constrained anisotropic constitutive model for the heterogeneous hyperelastic materials. Based on the sparse multiscale experimental data, the constitutive artificial neural networks for hyperelastic component phases containing composite interfaces are established by the particle swarm optimization algorithm. A microscopic finite element coupled constitutive artificial neural networks solver is introduced to obtain the homogenized stress-stretch relation of heterogeneous materials with different microstructures. And a dense stress-stretch relation dataset is generated by training a neural network through the FE results. Further, a generic invariant representation of strain energy function (SEF) is proposed with a parameter set being implicitly expressed by artificial neural networks (SANN), which describes the hyperelastic properties of heterogeneous materials with different microstructures. A convexity constraint is imposed on the SEF to ensure that the multiscale constitutive model is physically relevant, and the ℓ_1 regularization combined with thresholding is introduced to the loss function of SANN to improve the interpretability of this model. Finally, the multiscale model is hierarchically trained, cross-validated and tested using the experimental data of cord-rubber composite materials with different microstructures. The proposed multiscale model provides a convenient and general methodology for constitutive modeling of heterogeneous hyperelastic materials.

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1 Introduction

The heterogeneous materials composed of different hyperelastic phases, such as reinforced rubbers [1], soft biological materials [2] and solid rocket propellants [3], have abundant applications in engineering. Especially, the cord-rubber composites play an essential role in the tire industry, due to the advantages of high mobility, low noise and light weight and so on [4]. Complex microstructures, highly anisotropic properties, strong geometric and material nonlinearity lead to many difficulties in simulation of such heterogeneous materials in practical applications. Therefore, it is necessary to study the constitutive behaviors and mechanical performance of cord-rubber composites for the optimization design and the safety evaluation of tires.

Cord-rubber composites have been extensively researched over the past few decades, resulting in the development of various homogenized constitutive models derived from either phenomenological [5–8] or micromechanical [9–11] approaches. However, creating high-fidelity constitutive models for cord-rubber composites remains a significant research challenge for two primary reasons. First, design engineers might not have access to the substantial experimental characterization that existing phenomenological models typically desire beyond uniaxial tensile testing [8,12]. Second, phenomenological models are designed to describe the behaviour of cord-rubber composites with a specific microstructure [13], and they are unable to take into account changes in behaviour resulting from microstructural variations. Few phenomenological models have been able to accurately predict the mechanical behaviour of composites with different microstructures. Conversely, micromechanically inspired models are suitable for different microstructures within a limited parameter range, but they often fail to adequately describe measured data. Therefore, the existing constitutive modeling approaches are not suitable for the cord-rubber composites with various microstructures.

Data-driven model can effectively identify complex high-dimensional mapping relationships from stress-stretch data, thus becoming an important choice for constitutive modeling of composites [14–17]. The neural network-based surrogate model of composite constitutive relation is the most popular way. For example, artificial neural network [18–21] and Gaussian process regression model [22] are established based on the macroscopic strain-stress data to replace the constitutive relations of materials. Linka et al. [23] and Leng et al. [24,25] established the neural network-based relationship between microstructure descriptors and isotropic strain energy functions for rubber composites and biological tissues, respectively. Liu et al. [26] and Klein et al. [27] developed a physical informed neural network to model the anisotropic biomaterials. Vlassis et al. [28]