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High-Order Low-Dissipation Shock-Resolving TENO-THINC Schemes for Hyperbolic Conservation Laws

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Abstract. While the recently proposed TENO (targeted essentially non-oscillatory) schemes [Fu et al., Journal of Computational Physics 305 (2016): 333-359] exhibit better performance than the classical WENO (weighted essentially non-oscillatory) schemes with the same accuracy order, there is still a room for further improvement, e.g., the physical discontinuities may be significantly smeared by the excessive numerical dissipation due to the enforcement of the ENO property after a long-time advection. More recently, a new fifth-order TENO5-THINC scheme is proposed by coupling the TENO5 scheme with a non-polynomial THINC (tangent of hyperbola for interface capturing) scheme based on a parameter-free discontinuity indicator. The novelty originates from the fact that the new strategy locates the discontinuities accurately and deploys the jump-like THINC reconstruction scheme for resolving the discontinuities with a sub-cell resolution, instead of enforcing the ENO property. The new scheme successfully leverages the excellent wave-resolution property of standard TENO schemes for smooth and under-resolved continuous scales and the discontinuity-resolving capability of THINC for reconstructing genuine discontinuities. In this work, we further develop the low-dissipation discontinuity-resolving very-high-order TENO-THINC reconstruction schemes for hyperbolic conservation laws by proposing tailored coupling strategies. Without loss of generality, the six- and eight-point TENO-THINC schemes are developed, and the explicit formulas are given as well as the built-in parameters. Based on a set of critical benchmark simulations, the newly proposed schemes show

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significantly lower numerical dissipation when compared to the counterpart TENO schemes without sacrificing numerical robustness. The presented numerical results represent the state-of-the-art in the literature and can serve as references for future algorithm development.

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1 Introduction

Numerical simulations of high-speed compressible flows have a significant challenge due to the presence of both the discontinuities, such as shockwave and contact discontinuities and the high-wavenumber flow structures in the computational field. To avoid artificial numerical oscillations near discontinuity, lots of non-linear schemes have been developed, e.g., the TVD (total variation diminishing) [1], ENO (essentially non-oscillatory) [2], and WENO [3, 4] schemes. The ENO scheme selects the smoothest stencil from a set of candidate stencils and achieves the essentially non-oscillatory property near discontinuities. However, since the ENO scheme [2] always adopts only one candidate stencil from the full stencil both in the smooth regions and near the discontinuities, it cannot achieve the optimal accuracy order in a smooth solution compared to the counterpart linear scheme that has the same size of the full stencil. On the other hand, the WENO scheme [4] combines all candidate stencils with non-linear weights and can restore the desired convergence order in smooth regions. Numerical experiments demonstrate that the classical WENO5-JS [4] scheme fails to retain a fifth-order property near critical points. The improved versions of WENO family schemes have been studied to address these issues in recent years, e.g., the WENO5-M [5] and WENO5-Z [6] schemes. Other developments include, e.g., the WENO schemes with an optimized spectral property [7–9], the very-high-order WENO schemes [10–12], the hybrid WENO schemes [13–15], the central WENO (CWENO) schemes [16, 17], the WENO-AO [18] and WENO-ZQ [19] schemes, and etc. For a comprehensive review, the readers are referred to [20, 21].

Recently, a family of high-order TENO schemes with significantly improved performance has been proposed by Fu et al. [22–29]. Unlike WENO family schemes, TENO schemes deploy candidate stencils with incremental width in combination with a strong scale separation technique and a novel ENO-like stencil selection strategy. The standard TENO schemes of fifth- to eighth-order have been developed with spectral optimization [22, 23]. Numerical experiments suggest that the cutoff value C_T in the TENO weighting strategy is closely related to the magnitude of the nonlinear numerical dissipation. To better resolve the small-scale flow structures, a novel adaptation strategy, which adjusts the C_T value based on the local flow scales, is proposed [24, 25, 30] and the overall