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Numerical Approximation of an Axisymmetric Elastoacoustic Eigenvalue Problem

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Abstract. This paper deals with the numerical approximation of a pressure/displacement formulation of the elastoacoustic vibration problem in the axisymmetric case. We propose and analyze a discretization based on Lagrangian finite elements in the fluid and solid domains. We show that the scheme provides a correct approximation of the spectrum and prove quasi-optimal error estimates. We report numerical results to validate the proposed methodology for elastoacoustic vibrations.

AMS subject classifications: 65N25, 65N15, 74F10

Key words: Axisymmetric problem, fluid-structure interaction, spectral problem, finite element method.

1 Introduction

The aim of the paper is to introduce and analyze a finite element approximation for the axisymmetric elastoacoustic spectral problem. Fluid-structure interaction problems appears in several applications in two (2D) and three dimensional (3D) settings (propagation of noise, vibration of dams, off shore structures, seismic waves [13,21,28], etc.). We are interested in approximating axisymmetric solutions of the spectral problem. Such solutions appear when solving the 3D problem considering the case where both the computational domain and the parameters have axial symmetry, thus, it is possible to solve the 3D problem by a 2D one reducing the dimension and thereby the computational effort.

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We focus in studying the harmonic vibrations of an interacting elastic structure with a compressible fluid. In this case, the displacements are small and then we can assume a linear response of the structure. We neglect the effects of gravity and consider a homogeneous fluid for which its reference density is constant. We also assume other common simplifications in the fluid for these types of problems: that viscous effects are not relevant, and that the velocities are small enough to ignore the convective terms.

Regarding the numerical approximation of axisymmetric problems, different formulations and discretization have been introduced in recent years to approximate Darcy, Stokes, Brinkman and Elasticity axisymmetric problems (see, for instance, [1,6,7,18,27]). Also, axisymmetric coupled problems are studied in the context of the Stokes-Darcy problem in [19] and Navier-Stokes-Brinkman and transport equations in [5]. However, spectral approximation of axisymmetric problems has been only analyzed in the context of vibration modes of an acoustic fluid in [30,31] and, to the author's knowledge, the elastoacoustic spectral problem has not been studied in the axisymmetric setting.

Different formulations have been used to solve elastoacoustic problems in the cartesian setting, mainly in the frequency domain. While the vibrations of an elastic solid are generally described in terms of the displacement, different variables have been used for the fluid: pressure, displacement potential, displacements, velocity potential or combinations of some of them (see, for instance, [2,8,9,26,32,33]). One option is to choose the same primary variable than for the solid, the displacements, which leads to a symmetric formulation for the coupled problem. A spurious-free method is proposed in [8] which consists of using standard linear elements in the solid combined with Raviart-Thomas elements for the fluid displacement. Although this method works in the cartesian case, it is known from [30,31] that the same choice of finite elements introduces spurious eigenvalues interspersed among the actual ones of the axisymmetric acoustic problem. This spurious modes are associated to rotational displacements in the fluid. In order to avoid this drawback, in this paper we choose the pressure as primary variable in the fluid and the displacement in the solid (see, for instance, [9]). Such choice presents two important advantages: the interface conditions are build in into the variational problem and the pressure is a scalar magnitude which implies that rotational modes do not appear. Nevertheless, in the case of coupled systems, this approach results in a non-symmetric eigenvalue problem. In general, it is highly non-trivial to develop finite element methods for these problems (see [24,35]). First we show that the axisymmetric spectral problem is well defined and that its eigenvalues are real. For its numerical approximation we consider a finite element scheme based on Lagrangian elements and prove spectral convergence by using the theory of [22].

The outline of our paper is the following: In Section 2 the modeling equations are presented and a weak formulation for the 3D problem is introduced. The weighted functional spaces required for the analysis of the axisymmetric problem are introduced in Section 3. We also introduce a variational formulation for the axisymmetric elastoacoustic vibration problem and its corresponding spectral characterization. In Section 4, we introduce a finite element discretization of the coupled problem by using Lagrangian