

A Unified Gas-Kinetic Particle Method for Frequency-Dependent Radiative Transfer Equations with Isotropic Scattering Process on Unstructured Mesh

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Abstract. In this paper, we extend the unified gas kinetic particle (UGKP) method to the frequency-dependent radiative transfer equation with both absorption-emission and scattering processes. The extended UGKP method could capture the diffusion and free transport limit and provide a smooth transition in the physical and frequency space in the regime between the above two limits. The proposed scheme has the properties of asymptotic-preserving and regime-adaptive, which make it an accurate and efficient scheme in the simulation of multiscale photon transport problems. In the UGKP formulation of flux construction and distribution closure, the coefficients of the non-equilibrium free stream distribution and near-equilibrium Planck expansion are independent of the time step. Therefore, even with a large CFL number, the UGKP can preserve a physically consistent ratio of the non-equilibrium and the near-equilibrium proportion. The methodology of scheme construction is a coupled evolution of the macroscopic energy equation and the microscopic radiant intensity equation, where the numerical flux in the macroscopic energy equation and the closure in the microscopic radiant intensity equation are constructed based on the integral solution. Both numerical dissipation and computational complexity are well controlled, especially in the optically thick regime. 2D multi-thread code on a general unstructured mesh has been developed. Several numerical tests have been simulated to verify the numerical scheme and code, covering a wide range of flow regimes. The numerical scheme and code we developed are highly demanded and widely applicable in high-energy engineering applications.

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Key words: Frequency-dependent radiative transfer, multiscale method, asymptotic preserving, unified gas-kinetic particle method, unstructured mesh.

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1 Introduction

The thermal radiative transfer (TRT) equations, which describe the time evolution of radiative intensity and its interaction with the background material, have wide applications in astrophysics, atmospheric physics, inertial confinement fusion (ICF), high-temperature flow systems, plasma physics [1, 2], etc. It contains the kinetic radiation transport equation that describes the photon transport in the background material and the material energy equation that describes the energy exchange between radiation and background material. These two equations are coupled by the absorption-emission process that is characterized by the material opacity. The nonlinear dependency of the material opacity and material temperature makes the system difficult to solve [3, 4]. In addition, the high dimensionality of the equation greatly increases the computational cost. Developing numerical methods with high accuracy and high efficiency has become an important topic for the past decades.

Generally, the numerical methods for radiative transfer equations can be categorized into the deterministic method and the stochastic method. The deterministic methods include the macroscopic moment methods [5–10] and microscopic discrete ordinate SN method [11–15]. The moment methods propose a closure to the radiant intensity by expanding it in a specific functional space [16]. The SN methods directly discretize the velocity space using a specific quadrature. For stochastic methods, the most commonly used Monte Carlo (MC) method [17–20] exploits random numbers to simulate the interactions of individual radiation particles with the background material. The MC method is more efficient in optically thin regimes, especially for multi-dimensional cases, and does not suffer from the ray effect compared with the deterministic method. The implicit Monte Carlo (IMC) method proposed by Fleck and Cummings [17] is a popular Monte Carlo method for solving the TRT equations. This method approximates the rapid, dynamic timescale of photon absorption-emission processes via effective scattering events by the Fleck factor, according to which the nonlinear TRT equations are reformulated into a system of linearized equations and solved by the standard Monte Carlo method. However, it is generally noticed that the IMC method becomes inefficient in the optically thick region when the photon mean free path is much smaller than the flow characteristic length, and the particle collision becomes dominated. In such a regime, a great number of effective scattering events are calculated during a time step, which significantly increases the computational cost. Efforts have been made to improve the efficiency of the IMC method in optically thick regions [21, 22], such as the implicit Monte Carlo diffusion (IMD) [23], discrete diffusion Monte Carlo (DDMC) [24, 25] methods, as well as the moment-based scale-bridging method [26–30]. The IMD and DDMC methods are transport-diffusion hybrid methods that simulate the TRT equations with diffusion approximation in optically thick regions and the standard IMC method in other regions. For the transport-diffusion hybrid method, special efforts need to be made for the domain decomposition and the information exchange at transport-diffusion interfaces. For the moment-based scale-bridging method, coupled high-order and low-order