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# High-Order Adaptive Dissipation Scheme Based on Vortex Recognition for Compressible Turbulence Flow

Jiahong Cai, Shengye Wang* and Wei Liu

*College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, P.R. China*

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**Abstract.** In the numerical simulation of compressible turbulence involving shock waves, accurately capturing the intricate vortex structures and robustly computing the shock wave are imperative. Employing a high-order scheme with adaptive dissipation characteristics proves to be an efficient approach in distinguishing small-scale vortex structures with precision while capturing discontinuities. However, differentiating between small-scale vortex structures and discontinuities during calculations has been a key challenge. This paper introduces a high-order adaptive dissipation central-upwind weighted compact nonlinear scheme based on vortex recognition (named as WCNS-CU-$\Omega$), that is capable of physically distinguishing shock waves and small-scale vortex structures in the high wave number region by identifying vortices within the flow field, thereby enabling adaptive control of numerical dissipation for interpolation schemes. A variety of cases involving Euler, N-S even RANS equations are tested to verify the performance of the WCNS-CU-$\Omega$ scheme. It was found that this new scheme exhibits excellent small-scale resolution and robustness in capturing shock waves. As a result, it can be applied more broadly to numerical simulations of compressible turbulence.

**AMS subject classifications**: 35L65, 65M06, 76F50

**Key words**: Weighted compact nonlinear scheme, high-order, shock-capturing, compressible turbulence.

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## 1 Introduction

Compressible turbulent flow is a complex phenomenon that arises in various aerospace applications, such as supersonic aircraft, rocket engines, and scramjets. There will be various of complex flow structures, including shock waves, expansion waves, and boundary

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*Corresponding author. *Email addresses:* `wangshengye13@nudt.edu.cn` (S. Wang), `cjh_nudt@nudt.edu.cn` (J. Cai), `fishfather6525@sina.com` (W. Liu)

layer transition in compressible turbulent flow simultaneity. Understanding and predicting these flow phenomena is crucial for the design and optimization of high-speed aerospace systems. However, accurate modeling and simulation of compressible turbulent flow remain challenging tasks due to its inherent complexity.

In compressible turbulent flow, shock waves and vortices frequently coexist. Shock waves are physical discontinuities that necessitate adequate dissipation in numerical simulations to suppress numerical oscillations before and after the discontinuity. Conversely, the small-scale vortex structure requires minimal dissipation to prevent it from being smoothed out. Consequently, robustly capturing the shock wave while calculating the vortex structure with high fidelity simultaneously in numerical simulations is a formidable task in the simulation of compressible turbulent flow.

In order to capture the shock wave robustly in high-precision numerical simulations, Liu [1] proposed weighted essentially non-oscillatory (WENO), which is promoted by Jiang and Shu [2]. The WENO scheme is widely used in high-order numerical simulation of flows because of its excellent shock-capturing traits along with promising spectral resolution. However, the dissipative upwind biased discretization has an adverse effect on the simulation of scale resolution, especially in Large Eddy Simulation (LES) [3] and hybrid Reynolds-Averaged Navier-Stokes (RANS)/LES. Actually, schemes with superior dissipation characteristics, for example central schemes, perform better in LES with high Reynolds number [4].

For the purpose of reducing the influence of the upwind biased discretization, Hu [5] introduced a downwind stencil to WENO, resulting in the WENO-CU scheme, which is optimized for turbulence simulations, providing excellent resolution and dissipation properties. However, while WENO-CU exhibits low dissipation in smooth regions, its shock capturing ability is not entirely satisfactory. To address this issue, several central-upwind schemes have been developed, including WENO-CU6-M2 [6]. The main idea of most central-upwind schemes is to use upwind stencil in the discontinuous region to maintain the stability of the calculation, and use the central scheme in the smooth region of the flow field to reduce the numerical dissipation. Zhao [7] has also proposed an efficient adaptive central-upwind WENO-CU6 scheme (EWENO-CU6) to improve numerical robustness through a switch formula based on their shock sensor. Different from the mixed-type scheme by Hu, Zhao [8] also extends of even-order WENO schemes, which uses a common symmetrical stencil in the reconstruction of the variables at its both sides to suppress the spurious oscillation and improve the resolution in the region with discontinuities.

However, it is difficult for WENO to fulfill the property of the exact preservation of free-stream solutions on curvilinear meshes [9, 10], which is important in the simulation of vortex flow. To maintain the preservation of free-stream flow, it is crucial to satisfy the geometric conservation law (GCL), which can be achieved through the symmetrical conservative metric method (SCMM) [11]. Another well-known approach to capture discontinuities is the weighted compact nonlinear scheme(WCNS), developed by Deng [12]. While WCNS also employs the weighted approach, it separates the difference and inter-