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Thermal Regulation in Thin Vascular Systems: A Sensitivity Analysis

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Abstract. One of the ways natural and synthetic systems regulate temperature is via circulating fluids through vasculatures embedded within their bodies. Because of the flexibility and availability of proven fabrication techniques, vascular-based thermal regulation is attractive for thin microvascular systems. Although preliminary designs and experiments demonstrate the feasibility of thermal modulation by pushing fluid through embedded micro-vasculatures, one has yet to optimize the performance before translating the concept into real-world applications. It will be beneficial to know how two vital design variables—host material's thermal conductivity and fluid's heat capacity rate—affect a thermal regulation system's performance, quantified in terms of the mean surface temperature. This paper fills the remarked inadequacy by performing adjoint-based sensitivity analysis and unravels a surprising non-monotonic trend. Increasing thermal conductivity can either increase or decrease the mean surface temperature; the increase happens if countercurrent heat exchange—transfer of heat from one segment of the vasculature to another—is significant. In contrast, increasing the heat capacity rate will invariably lower the mean surface temperature, for which we provide mathematical proof. The reported results (a) dispose of some misunderstandings in the literature, especially on the effect of the host material's thermal conductivity, (b) reveal the role of countercurrent heat exchange in altering the effects of design variables, and (c) guide designers to realize efficient microvascular active-cooling systems. The analysis and findings will advance the field of thermal regulation both on theoretical and practical fronts.

AMS subject classifications: 35Q79, 90C31, 65N30

Key words: Sensitivity analysis, adjoint state method, thermal regulation, microvascular systems, active cooling, countercurrent heat exchange.

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A list of mathematical symbols and abbreviations

Symbol	Definition
	Operators
$[\![\cdot]\!]$	jump operator across the vasculature Σ
$\langle\!\langle\cdot\rangle\!\rangle$	average operator across the vasculature Σ
$\mathrm{div}[\cdot]$	spatial divergence operator
$\mathrm{grad}[\cdot]$	spatial gradient operator
	Geometry-related quantities
Ω	domain (i.e., mid-surface of the body)
$\partial\Omega$	boundary of the domain
Γ^{ϑ}	part of the boundary with prescribed temperature
Γ^{q}	part of the boundary with prescribed heat flux
Σ	curve representing the vasculature
d	thickness of the body
$\widehat{\mathbf{n}}(\mathbf{x})$	unit outward normal vector to the boundary
$\widehat{\mathbf{n}}^{\pm}(\mathbf{x})$	unit outward normal vector on either sides of Σ
s	normalized arc-length along Σ, measured from the inlet under forward flow
$\widehat{\mathbf{t}}(\mathbf{x})$	unit tangential vector along Σ
$\mathbf{x}$	a spatial point
	Solution fields
$\vartheta(\mathbf{x})$	temperature (scalar) field
$\vartheta^{(f)}(\mathbf{x})$	temperature field under forward flow conditions
$\vartheta^{(r)}(\mathbf{x})$	temperature field under reverse flow conditions
$\vartheta_{\mathrm{outlet}}$	outlet temperature
$\mathbf{q}(\mathbf{x})$	heat flux vector field
	Prescribed quantities
ϑ_{amb}	ambient temperature
$\vartheta_{\mathrm{inlet}}$	inlet temperature
$f(\mathbf{x})$	power supplied by heat source
f_0	a constant power by heat source
Q	volumetric flow rate in the vasculature
	Material and surface properties
κ	host material's thermal conductivity
ρ_f	fluid's density
c_f	fluid's specific heat capacity
h_T	(combined) heat transfer coefficient
	Other symbols
η	thermal efficiency
$\overline{\vartheta}$	spatial mean of the temperature field (i.e., mean surface temperature)