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An Implicit, Asymptotic-Preserving and Energy-Charge-Conserving Method for the Vlasov-Maxwell System Near Quasi-Neutrality

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Abstract. An implicit, asymptotic-preserving and energy-charge-conserving (APECC) Particle-In-Cell (PIC) method is proposed to solve the Vlasov-Maxwell (VM) equations in the quasi-neutral regime. Charge conservation is enforced by particle orbital averaging and fixed sub-time steps. The truncation error depending on the number of sub-time steps is further analyzed. The temporal discretization is chosen by the Crank-Nicolson method to conserve the discrete energy exactly. The key step in the asymptotic-preserving iteration for the nonlinear system is based on a decomposition of the current density deduced from the Vlasov equation in the source of the Maxwell model. Moreover, we show that the convergence is independent of the quasi-neutral parameter. Extensive numerical experiments show that the proposed method can achieve asymptotic preservation and energy-charge conservation.

AMS subject classifications: 65M06, 78A25, 78A35, 78-08

Key words: Vlasov-Maxwell, quasi-neutrality, asymptotic-preserving, energy-charge conservation.

1 Introduction

The Vlasov-Maxwell (VM) system is of great importance in the modeling of collisionless magnetized plasmas, with a wide range of applications to fusion devices, high-power microwave generators, and large-scale particle accelerators. The VM system is a coupling of a kinetic equation and a field equation, in which the Vlasov equation describes the motion of microscopic particles, while the electromagnetic field is a solution of the Maxwell equations coupled to the Vlasov equations through the electrical charge and current.

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After scaling, the dimensionless VM system (see Section 2) depends on the scaled Debye length λ. It is the ratio of the physical Debye length λ_D, which is the distance traveled by a particle at thermal velocity in $1/2\pi$ of a plasma cycle, to a spatial scale, and is related to the plasma frequency [1, 6]. In the scaled model, the parameter λ appears in the Maxwell-Ampère equation and in Gauss's law. The electric field cannot be obtained explicitly due to the singular nature of the quasi-neutral limit. In addition, the scaled Debye length λ controls the temporal and spatial frequencies of plasma oscillations and electromagnetic waves, which may become large as λ goes to zero. Therefore, the classical explicit scheme enforces small mesh sizes and time steps to resolve the quasi-neutral parameter. More challenging is the fact that the parameter may vary by orders of magnitude over time and space, which makes traditional domain decomposition methods [18, 19, 22, 35] impossible. A good choice is the Asymptotic-Preserving (AP) scheme, first coined in [29], which switches from a microscopic solver to the macroscopic solver automatically. For more representative AP schemes, we refer to [30]. In the literature, a wide range of AP schemes have been developed for various plasma models in the quasi-neutral regime, including the Euler-Poisson [14], Euler-Maxwell [17], and Vlasov-Poisson systems [16,25,26]. For the Vlasov-Maxwell, Degond et al. developed an AP scheme [15] by reformulating the VM system to unify the quasi-neutral model and non-neutral model in a single set of equations.

The VM system itself is energy-charge-conserving. However, all the aforementioned AP schemes do not conserve the total discrete energy. Numerical noise introduces spurious energies that can erroneously feed plasma instabilities, leading to unphysical results. In the study of plasma simulations, it is essential to observe the transformation of energy from one component to another. Most energy-conserving methods are based on implicit methods [7, 9–11, 31, 32]. They relax the time-step constraint for stability and have good properties for long-time computation.

Clearly, all the fully implicit methods are consistent with both non-neutral and quasi-neutral models. However, a fully implicit discretization requires the solution of a nonlinear system. The convergence of the iterative algorithm is severely affected by the small parameter λ. The iteration does not even converge as λ goes to zero. This is because the iterative procedure is usually based on a linearized approach, which leads to the enforcement of some nonlinear constraints depending on λ. There are not many studies that satisfy both energy-charge conservation and asymptotic preservation. Most recently, Ji et al. proposed an Asymptotic-Preserving and energy-conserving (APEC) scheme [28] based on the AP scheme through a Lagrange multiplier to correct the kinetic energy. The goal of this paper is to design, analyze and validate an implicit Asymptotic-Preserving and energy-charge-conserving (APECC) Particle-In-Cell (PIC) method for VM system of plasma physics near quasi-neutrality. Our AP methodology is partly motivated by the work of Filbet and Jin [21], which is applied to physical problems with stiff source terms that admit stable and unique local equilibrium. However, their method was not aimed at nonlinear iterations.

The contributions of this work lie in three aspects.