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A Model-Data Asymptotic-Preserving Neural Network Method Based on Micro-Macro Decomposition for Gray Radiative Transfer Equations

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Abstract. We propose a model-data asymptotic-preserving neural network(MD-APNN) method to solve the nonlinear gray radiative transfer equations (GRTEs). The system is challenging to be simulated with both the traditional numerical schemes and the vanilla physics-informed neural networks (PINNs) due to the multiscale characteristics. Under the framework of PINNs, we employ a micro-macro decomposition technique to construct a new asymptotic-preserving (AP) loss function, which includes the residual of the governing equations in the micro-macro coupled form, the initial and boundary conditions with additional diffusion limit information, the conservation laws, and a few labeled data. A convergence analysis is performed for the proposed method, and a number of numerical examples are presented to illustrate the efficiency of MD-APNNs, and particularly, the importance of the AP property in the neural networks for the diffusion dominating problems. The numerical results indicate that MD-APNNs lead to a better performance than APNNs or pure Data-driven networks in the simulation of the nonlinear non-stationary GRTEs.

AMS subject classifications: 35Q49, 65M12, 68T07, 80A21

Key words: Gray radiative transfer equation, micro-macro decomposition, model-data, asymptotic-preserving neural network, convergence analysis.

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1 Introduction

The gray radiative transfer equations (GRTEs) describe the radiation transport of photons and energy exchange with background materials, which find a wide range of applications in the fields of astrophysics, inertial/magnetic confinement fusion, high-temperature flow systems, and so on [1–3]. As a type of kinetic model coupled with a nonlinear material thermal energy equation, an accurate simulation of GRTEs is nontrivial to achieve because of the high dimensionality, strongly coupled nonlinearity, and multi-scale features caused by different opacities of the background materials.

The stochastic and deterministic methods are two widely applied conventional numerical approaches for the GRTEs. The former, for instance the Implicit Monte Carlo (IMC) method [4–9], has the advantage on dealing with the high dimensionality, and however the convergence rate is low and suffers statistical noises. While the deterministic schemes (e.g. finite difference/element/volume methods, discontinuous Galerkin method) with the asymptotic preserving (AP) technique are designed to capture the multi-scale features and treat the coupled nonlinearity. However, for high-dimensional cases, the discretization of the deterministic methods yields a very large algebraic system to be solved, which demands huge computational costs. The AP schemes for the neutron transport problems were first proposed by Larsen and Keller [10], Larsen [11–13], and then further developed by Larsen, Morel and Miller [14], Larsen and Morel [15], Jin and Levermore [16, 17]. For the unsteady cases, the AP schemes are constructed based on a decomposition of the distribution into the equilibrium and disturbance parts [18, 19]. The method has been further developed and extended to the multi-scale kinetic equations [20–22], and to the GRTEs, for example the AP unified gas kinetic scheme (AP-UGKS) [23, 24], the AP filtered PN method (PPFP$_N$-based UGKS) [25], the high order AP discontinuous Galerkin (AP-DG) method based on micro-macro decomposition [27] and the AP IMEX (AP-IMEX) method based on P_N decomposition [28].

In recent years, the deep learning method has become a competitive method for solving partial differential equations (PDEs). The idea is to approximate the unknown using the deep neural network by constructing suitable loss functions for optimization. Various neural network methods have been proposed, such as the physics-informed neural network (PINN) method [30] and the deep Galerkin method (DGM) [31] with the loss established on the L^2-residual of the PDEs, and the deep Ritz method (DRM) [32] with the loss based on the Ritz formulation, and so on. For other methods, we refer the reader to [33–36].

The PINNs and Model-Operator-Data Network (MOD-Net) have been applied to solve steady and unsteady linear radiative transfer models [34,37–41]. But the deep learning method for the nonlinear time-dependent GRTEs has not been fully investigated. We find that for GRTEs, the PINN loss function deteriorates in diffusion regimes with relatively small scale parameters ϵ, since PINNs tend to learn simplified models during the training process and lose the accuracy of asymptotic limit states. To tackle this problem, designing an asymptotic-preserving loss function is necessary [41].